A short recall about the history of the concept of refractive index

(From the 2nd to the 19th century)

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Introduction

- While the law of refraction was studied by **Ptolemy**, **Ibn Sahl**, **Ibn Al-Haytham** (**Alhazen**), **Kepler**, **Harriot**, **Snellius**, **Descartes**, **Fermat**, **Huygens** and **Newton**, it was not until the early nineteenth century, thanks to researches on colors and achromatic lenses, that the concept of refractive index appears.

- It is indeed since **Young** ("*refractive index*") and **Fraunhofer** ("*Exponent des Brechungsverhältnisses*", symbol ***n***) that refraction has been explicitly linked to a characteristic property of the transparent medium studied.

- We propose here a brief (and very partial) history of conceptual developments that have led to the "*index*" of refraction.

Ptolemy : An underestimated optician

- **Ptolemy**'s <u>Optics</u> was discovered in Europe by a XIIth century Latin translation of a lost Arabic partial translation of the lost Greek original manuscript... But its distribution remained limited, overshadowed by **Alhazen**'s <u>Optics</u>.

- Nowadays, it remains largely unknown despite of two modern-language translations (and commentaries) available in French (**Albert Lejeune**) and English (**A. Mark Smith**).

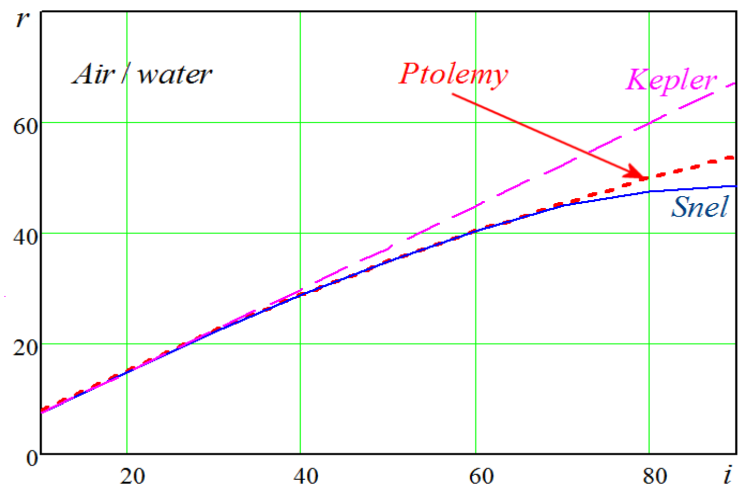
- **Ptolemy**'s work is a continuation of **Euclid**'s. It follows the Pythagorean theory of the extramissionist "***visual ray***" (coming from the eye).

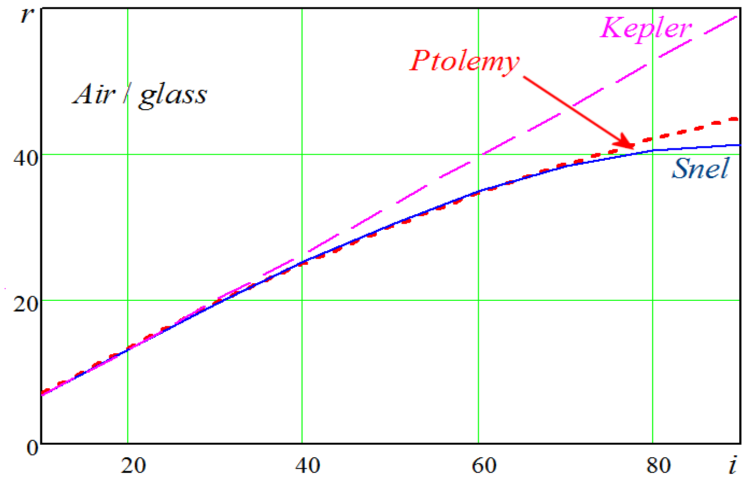
- However for geometrical optics, it brings deeper explanations and demonstrations, offers many ***experiments*** and goes further than Euclid's basic theorems. In the fifth book, it discusses ***refraction***.

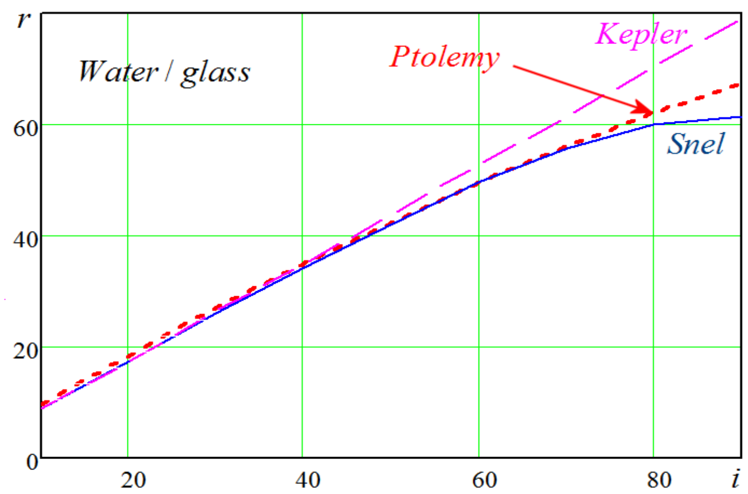
Ptolemy : the "experimentalist"

- To quantify the relationship between the incident angle $\boldsymbol{i}$ and the refracted angle $\boldsymbol{r}$ at the air-water, air-glass and water-glass interfaces, **Ptolemy** describes three *experiments* and provides three tables of "*experimental*" values:

v	i_v	r_v air/water	r_v air/glass	r_v water/glass
1	10	8	7	9.5
2	20	15.5	13.5	18.5
3	30	22.5	19.5	27
4	40	29	25	35
5	50	35	30	42.5
6	60	40.5	34.5	49.5
7	70	45.5	38.5	56
8	80	50	42	62







→ In all three cases up to 70°, the experimental "measurements" seem close to the values given by Snell's law

→ However, Ptolemy's results are obviously "adjusted" so that they coincide with a series (which is known to have been applied to astronomical calculations by the late Babylonians).

→ In Ptolemy's tables, a linear progression of ***r*** is countered by a regression of half a degree for every 10° increment of ***i*** (constant second difference).

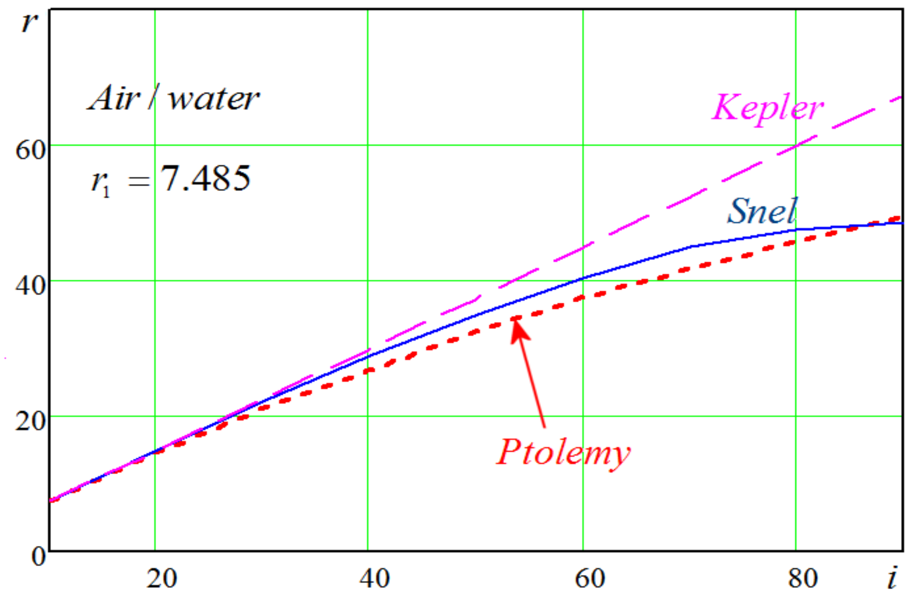
Ptolemy : the belief that a law exists

Ptolemy's algorithm amounts to consider a sequence which is strongly dependent on the first value r_1 of the refraction angle.

$$r_v = vr_1 - \frac{v(v-1)}{2} \times 0.5$$

- Ptolemy systematically overestimated (by about half a degree) the first value of r_1 in all three cases. If he had considered r_1 values closer to those of Snell's law, the sequence would have been significantly deviated from his other experimental values.

Example of the Air/water refraction



- As noted by **A. Mark Smith**, due to the limitations of the Euclidean theory of proportionality, **Ptolemy** would not have been able to express his sequence in an algebraic form.

- His "cheating" of half a degree may also be a reason why **Ptolemy** did not give his sequence explicitly. Anyway, the hidden presence of the latter shows that **Ptolemy** believed in the existence of a *mathematical* **law of refraction**.

- Above all, **Ptolemy** claimed that, contrary to what it seems, some equivalent to the equality of angles that occurs in mirrors must also be "*conserved*" in the refraction, due to "*the course of nature in conserving the exercise of power*."

- **Ptolemy** finally tried to develop a *physical* interpretation of refraction. So, he connected the refraction of the "*visual ray*" to the "*difference in **density** between the two medias*" considered. This explanation will remain the most commonly adopted until modern times.

The discoverers? : from Ibn Sahl to Descartes

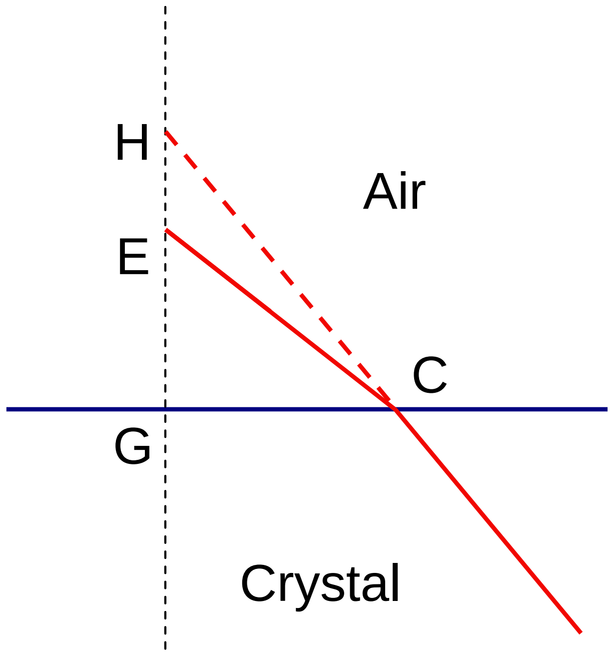
*At least four scientists are supposed to have "discovered" the exact law of refraction: **Ibn Sahl** in the 10th century, **Harriot**, **Snell** and **Descartes** in the 17th century. **Kepler**, who stated the law for small angles in 1611, could be added to this list.*

- At the end of the 1980's, Roshdi Rashed (CNRS-Paris) reconstructed the <u>Treatise On Burning Instruments</u> of the Baghdadi scientist **Ibn Sahl**.
- In his work, written around 984, this pure mathematician used the <u>Optics</u> of **Ptolemy** (in particular the result that the plane formed by the incident, reflected or refracted rays is perpendicular to the surface of reflection or refraction) and the *theory of conics* as well.

- **Ibn Sahl** used the ratio CE/CH<1 as a constant to show that parallel light beams crossing hyperboloidal lens surfaces converge at a *burning point*.

- This constancy of the ratio for a crystal is equi-valent to Snell's law: **CH/CE = sin*i* / sin*r* = *n***.

- However, **Ibn Sahl** did not:
  - present any experimental data;
  - explain how or where he got this result.



- Possibly, he first assumed that converging lenses must be limited by hyperboloidal surfaces and then deduced the constancy of the ratio.

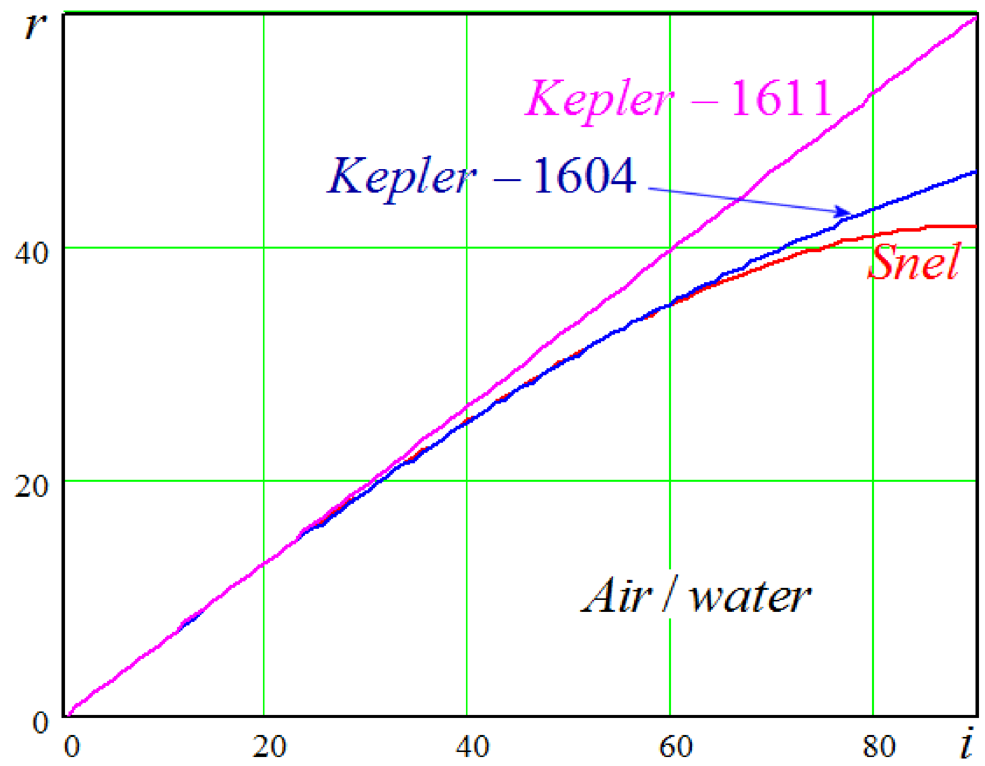
- Probably, as he was only interested in the *theory of burning glasses*, he did not realize how the constancy of CH/CE is a *general law* of geometrical optics and a *property* (the refractive index !) of crystal.

- **Alhazen**, who was acquainted with **Ibn Sahl**'s writings, did not mention or use his result about refraction.

- In his Ad Vitellionem Paralipomena (1604), **Kepler** studied the Tables of Ptolemy reproduced by Witelo.

- Like **Ibn Sahl**, **Kepler** was interested in anaclastics. More circumspect than **Witelo** towards **Ptolemy**'s underlying algorithm, he developed a complex reasoning that led to the law:

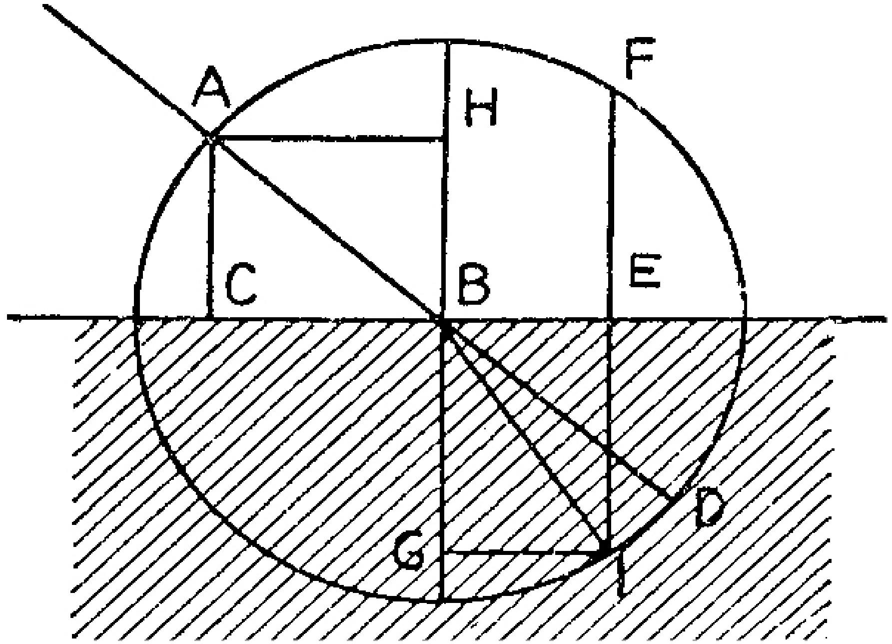
$$i = \frac{r}{1 - \dfrac{k}{\cos r}} \qquad \left(k = 1 - \frac{1}{n}\right)$$



- This law was not perfect, especially beyond 60°. In 1611, Kepler preferred to present a simpler law: ***the deviations are approximately proportional to the incidences lower than* 30° *angles*.**

- At the beginning of the XVIIth century, two scientists, **Thomas Harriot** (in 1602) and **Willebrord Snell** (in 1621) rediscovered the law that **Ibn Sahl** failed to transmit. They did not publish this law before their death and it was finally **Descartes** who first published it in 1637 (Dioptrique).

« la raison ou proportion […] qui est entre les lignes AH et IG ou semblables, demeure la même en toutes les réfractions qui sont causées par les mêmes corps. » (*2^{e} discours*)



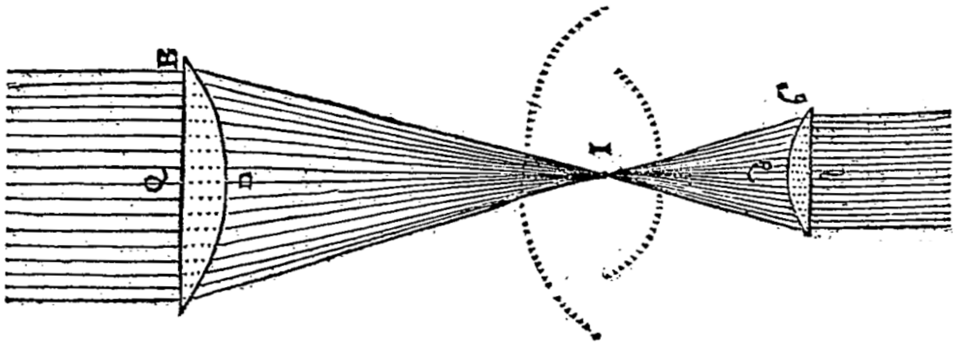
$$\frac{AH}{IG} = \text{constant} = f\left(\begin{array}{c} \textit{nature of the} \\ \textit{transparent media} \end{array}\right)$$

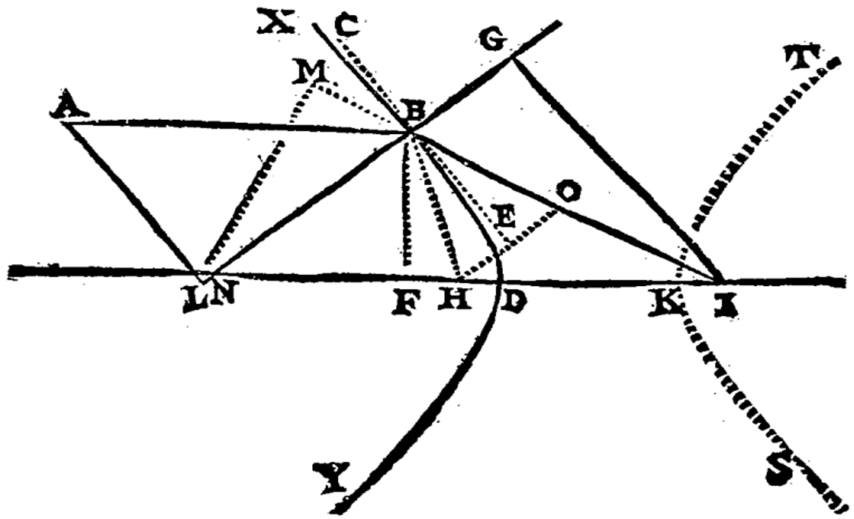
- For us, if the vacuum “fills” the space over the plane CBE, the “ratio” AH/IG is the refractive index. However for **Descartes**, the vacuum did not exist. Therefore, he was not able to conceive a “ratio” independent of the relation between the two media of the experiment.

- **Descartes** justified the law of refraction by an analogy with the trajectory of a bullet through a sieve. He was wrong in concluding that light goes faster in a dense medium (later, **Newton** did the same mistake).

- Nevertheless, according to **Leibniz** (and **Huygens**), there is “*lieu de soupçonner qu’il* [Descartes] *ne l’auroit jamais trouvée* [the law] *par là, s’il n’avoit rien appris en Hollande de la découverte de Snellius*” ([Discours de Métaphysique])

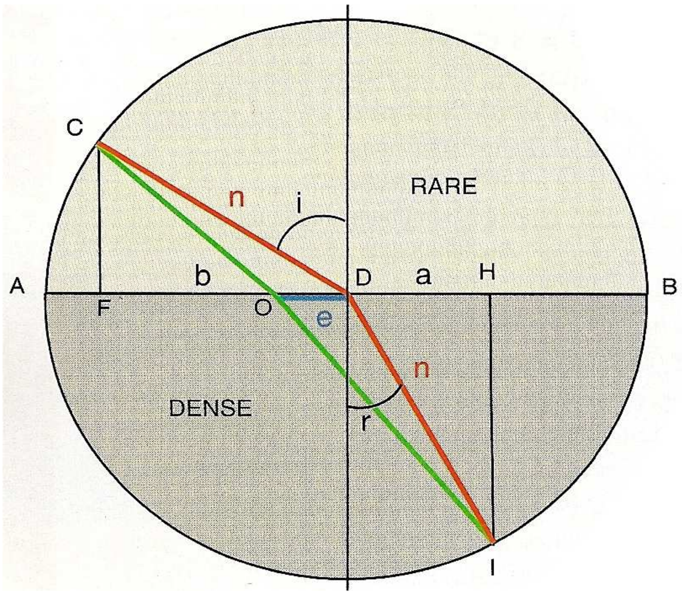
- Indeed, later in the [Dioptrique] (*8e discours*), the law of refraction “justifies” the use of conic surfaces for converging lenses, in the “manner” of **Ibn Sahl**.

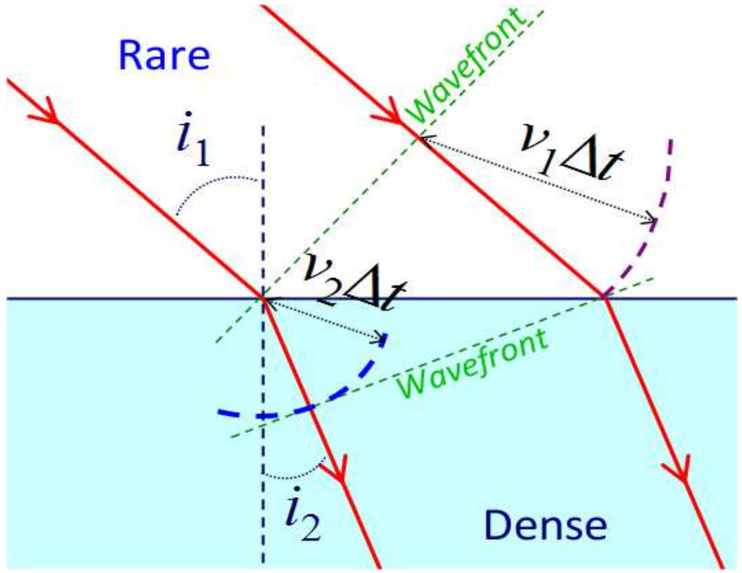




Fermat and Huygens: Light follows the fastest

- A better understanding of refraction is due to **Fermat** and **Huygens**.
- Both assumed that, contrary to **Descartes**'s and **Newton**'s opinions, light velocity is smaller in a dense medium than in a rare one.
- **Fermat** showed it by rediscovering **Snell**'s law. The time interval between two distant points of two different media must be minimized (1658).
- **Huygens** showed it by making the hypothesis that light is a wave (1673).

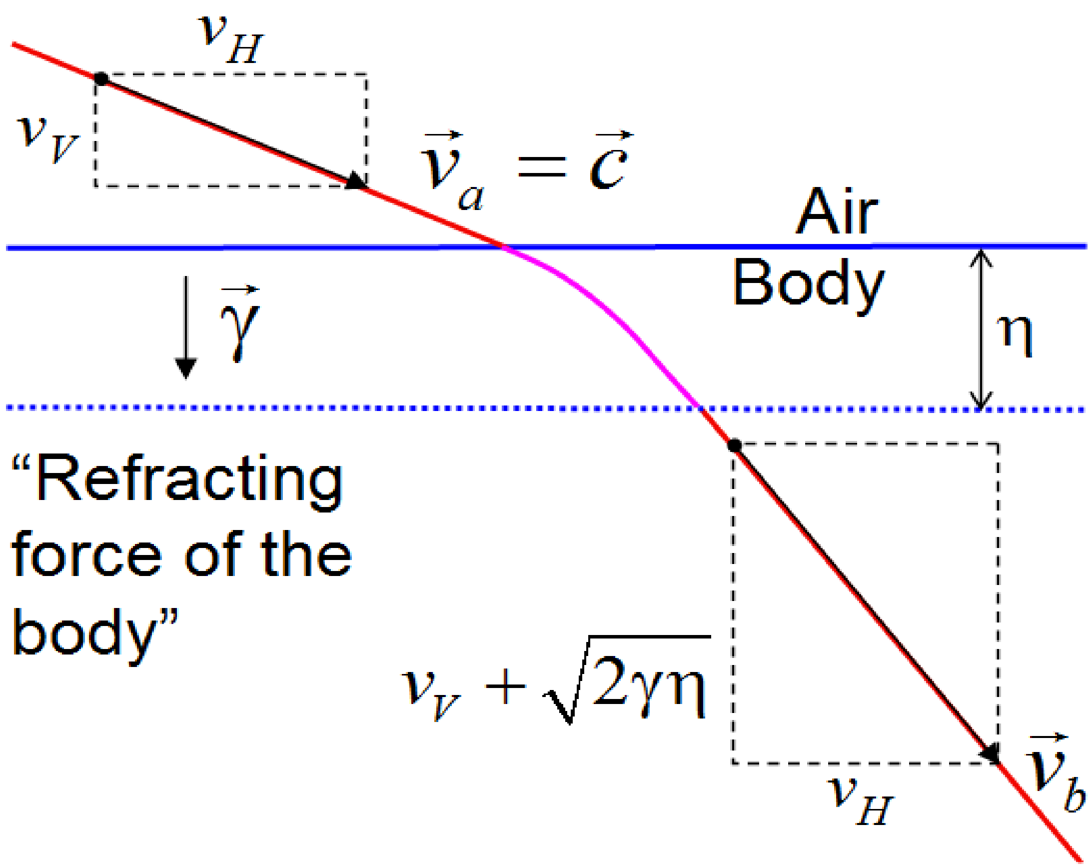




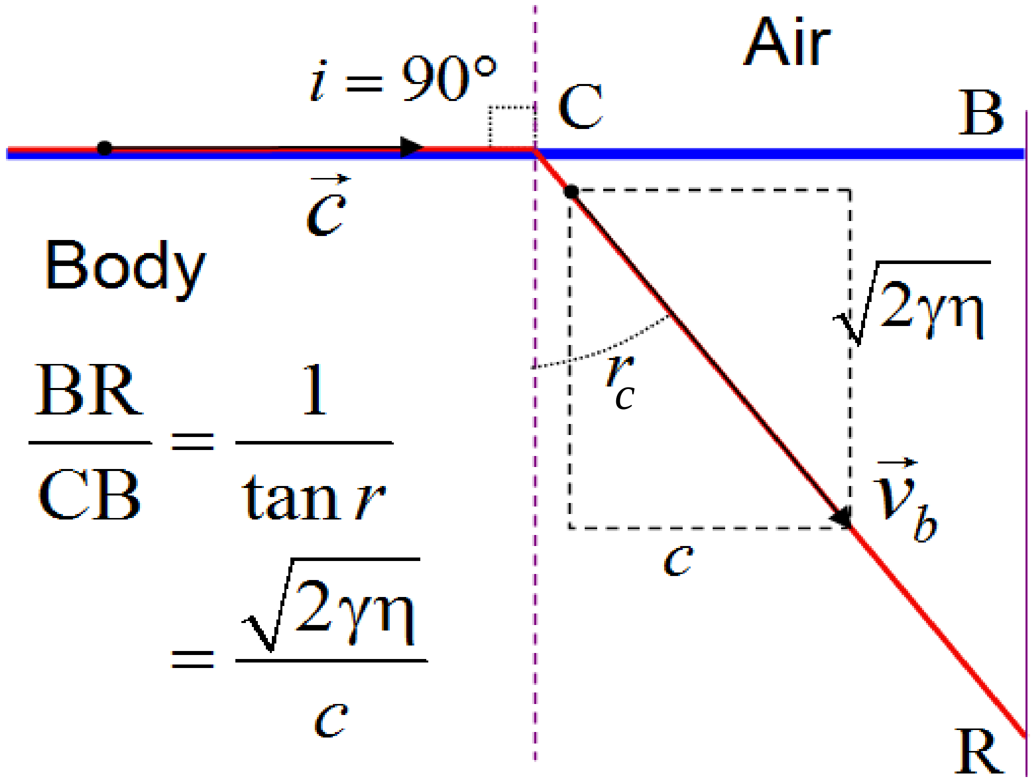
Newton: Towards a definition of the index

- In optics, the main refraction study of **Newton** is about the decomposi-tion of white light into the colours of the spectrum. But he was wrong in deducing from it that any type of lens would suffer from chromatic aberration.

- In addition, **Newton** was mainly in favor of a particle theory of light. He explained refraction in a dense medium by the existence of a refractive power in a thin layer of the medium (in the vicinity of its surface).



- From its mechanistic theory, **Newton** proposed to calculate the ***refractive power*** considering r_c, the critical angle of refraction (for a horizontal low-angled incident ray).



$$\frac{\mathrm{BR}}{\mathrm{CB}} = \frac{1}{\tan r} = \frac{\sqrt{2\gamma\eta}}{c}$$

$$\text{Refractive Power} = \gamma$$

$$\gamma \propto BR^2 = [\cot r_c]^2$$

- **Newton** then ranked various diaphanous materials depending on the ratio of the ***refractive power*** to the ***density***.

The refracting Bodies.	The Proportion of the Sines of Incidence and Refraction of yellow Light.	The Square of BR, to which the refracting force of the Body is proportionate.	The density and specifick gravity of the Body.	The refractive Power of the Body in respect of its density.
Pseudo-Topazius	23 to 14	1'699	4'27	3979
Air.	**3201 to 3200**	**0'000625**	**0'0012**	**5208**
Glass of Antimony.	17 to 9	2'568	5'28	4864
A Selenitis.	61 to 41	1'213	2'252	5386
Glass vulgar.	31 to 20	1'4025	2'58	5436
Crystal of the Rock.	25 to 16	1'445	2'65	5450
Island Crystal.	5 to 3	1'778	2'72	6536
Sal Gemmæ.	17 to 11	1'388	2'143	6477
Alume.	35 to 24	1'1267	1'714	6570
Borax.	22 to 15	1'1511	1'714	6716
Niter.	32 to 21	1'345	1'9	7079
Dantzick Vitriol.	303 to 200	1'295	1'715	7551
Oil of Vitriol.	10 to 7	1'041	1'7	6124
Rain Water.	**529 to 396**	**0'7845**	**1'**	**7845**
Gum Arabick.	31 to 21	1'179	1'375	8574
Spirit of Wine.	100 to 73	0'8765	0'866	10121
Camphire.	3 to 2	1'25	0'996	12551
Oil Olive.	22 to 15	1'1511	0'913	12607
Linseed Oil.	40 to 27	1'1948	0'932	12819
Spirit of Turpentine.	25 to 17	1'1626	0'874	13222
Amber.	14 to 9	1'42	1'04	13654
A Diamond.	100 to 41	4'949	3'4	14556

"Terrestrial stony alcalizate concretes"

"Middle degree"

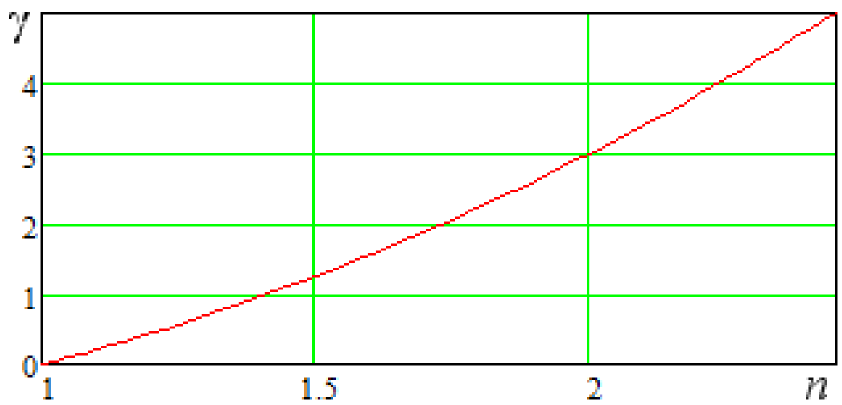
"Unctuous Bodies"

- First, **Newton** considered the refractive power of the ***air*** independently ("*determined by that of the Atmosphere observed by Astronomers*"). Clearly, Newton measured refractive powers with respect to ***vacuum***.

- Second, beyond the chemical body ranking, it should be noted that Newton tried to assign the body a *characteristic property*, the refractive power, which is directly related to what we now call the refractive index.

$$\gamma \propto BR^2 = [\cot r_c]^2$$

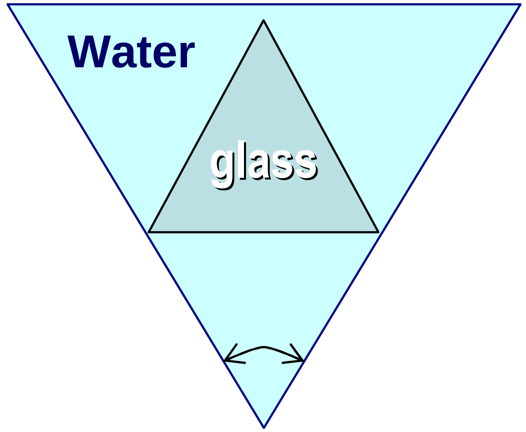
$$\Rightarrow \quad \gamma \propto \left[\cot\left(\arcsin\frac{1}{n}\right)\right]^2$$



- Finally, despite of his wrong mechanistic approach, Newton was the first who tried to connect quantitatively the refractive properties with the chemical composition of the bodies (and not only with their densities).

The revolution of the achromatic lenses

- In 1747, **Euler**, referring to the existence of the eye (free of chromatic aberration), showed theoretically it is possible to eliminate aberration in a set of lenses.



- In 1755, the Swedish **Klingenstierna**, in an experiment involving a glass prism and a variable-angle prism of water, showed that the dispersion can be removed without eliminating the deflection of the beam, and vice versa.

- In 1757, **John Dollond**, knowing the works of **Euler** and **Klingenstierna**, made the “first” achromatic objective* by combining a convex crown glass and a concave flint glass lenses. He obtained the exclusive right to manufacture achromatic sets of lenses until 1772.

* *It seems that such an objective has been already made by **Hall** and **Bass** around 1733.*

- In his paper, **Dollond** continued to consider the ratio of the sinuses instead of a characteristic quantity for each material used.

- However, with the interest in achromatic lenses, the need to consider a quantity that characterizes simply each diaphanous material was made more imperative.

- In his <u>Lectures</u> of 1807, **Thomas Young** refers skillfully to **Newton** in order to introduce a simplified measurement of "*refractive power*".

- First (p.413), **Young** recalled that the *"refractive power"* of a medium is measured by considering its interface with air and that air has *"the same sensible effect as a vacuum or an empty space"*.

- Finally, **Young** asserted that *"the ratio of the* [...] *sines* [can be] *expressed by the ratio of 1 to a certain number, which is called **the index of the refractive density of the medium**"* and *"that the **index of refraction** at the common surface of any two mediums is the quotient of their respective indices"*.

- In 1814, **Fraunhofer** used the same ratio of the sinuses in his work Bestimmung des Brechungs- und Farbenzerstreuungs- Vermögens verschiedener Glas-arten, in Bezug auf die Vervollkommnung achromatischer Fernröhre (Determination of Refractive Dispersion of various colors and types of glass, with respect to the perfection of achromatic telescopes).

- **Fraunhofer** gave to this ratio the terminology *„Exponent des Brechungs-Verhältnisses“* (strictly equivalent to **Young**’s) and the symbol ***n***.

Brechende Mittel	Exponenten der Brechungs-Verhältnisse.					
	On	Nn	Mn	Ln	Kn	In
Flintglas Nro. 13	1,63074	1,63505	1,63933	1,64349	1,64775	1,65203
Crownglas Nr. 9	1,52736	1,52959	1,53173	1,53380	1,53586	1,53783
Wasser	1,33209	1,33359	1,33501	1,33635	1,33763	1,33888

Conclusion

- The research of a refraction law played a major role in the development of the optics since the first attempts of **Ptolemy** until the more accomplished results of **Ibn Sahl**, **Kepler**, **Snell** or **Descartes**.

- However, it was necessary to wait for the beginning of the XIXth century, much later than the theory of colours of **Newton** and thanks to

  - **Newton**'s wrong mechanistic theory of refraction,

  - researches on the achromatic glasses,

  so that emerged the concept of refractive index and so that it begins to be understood well.

- The slideshow was a very beginning of a work about the history of the concept of refractive index from Antiquity till now. We are living today what will be tomorrow the history of nonlinear refractive index…

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