

NONLINEAR OPTICS OF INTENSE FEW-CYCLE PULSES: AN OVERVIEW  
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We provide a brief overview of recent theoretical and experimental studies in the field of nonlinear optics of intense few-cycle pulses, which were performed in a series of relevant physical settings.

*Key words:* few-cycle pulses, extreme nonlinear optics, attosecond physics, few-cycle solitons.

## 1. INTRODUCTION

The current use of ultrashort (few-cycle) intense laser pulses extends the capability of observing and controlling many physical phenomena, such as electron motion in atoms, molecular vibrations, and chemical reactions of complex molecules. Additionally, there exist a wealth of applications, ranging from testing ultrahigh-speed semiconductor devices to precision processing of materials, and to sophisticated surgical operations in ophthalmology and neurosurgery [1]. An intense research activity in the field followed seminal results, reported in 1999, on the experimental generation and characterization of ultrashort (two-cycle or even sub-two-cycle) optical pulses from Kerr-lens mode-locked Ti:sapphire lasers [2]–[5]. Such ultrashort optical pulses can be used in a variety of applications ranging from light-matter interactions, high-order harmonic generation, extreme nonlinear optics [6], and to attosecond physics [7, 8].

Today, several methods for the amplification of laser pulses are available, thus enabling the investigation of new phenomena in the area of nonlinear optics. A first method, the so-called *chirped-pulse amplification* (CPA) introduced by Strickland and Mourou [9], allows for the amplification of laser pulses with focused peak intensities in the range of  $10^{22}$  W/cm<sup>2</sup>. In fact, CPA is a powerful method for amplifying ultrashort laser pulses up to the petawatt (PW) level, upon stretching out the laser pulses temporally and spectrally prior to amplification. After the invention of CPA, another important advancement in the generation of ultrashort laser pulses has been the *optical parametric chirped pulse amplification*, introduced by Dubietis *et al.* in 1992 [10]. A third amplification technique, introduced by Malkin *et al.* in 1999 [11], relies on a new compression method based on *backwards Raman scattering* and has the advantage of avoiding the use of diffraction gratings. In 2012 Mourou *et al.* [12] put forward a fourth amplification technique, the so-called *cascaded conversion compression*, which has the capability to compress – with good efficiency – nanosecond laser pulses of energies about 10 kJ into femtosecond pulses having the same energy. By combining the above mentioned techniques or/and by developing others, it is naturally expected that exawatt-zettawatt peak powers might soon be reachable. Notice that the possibility of generating zeptosecond and even yoctosecond electromagnetic pulses, thus allowing for operation on nuclear as well as quark-gluon plasma time scales, was recently investigated by Kaplan [13].

The aim of this work is to provide a brief overview of recent experimental and theoretical advances in the field of nonlinear optics of intense few-cycle pulses, and also in the area of few-cycle optical solitons arising in a variety of physical settings. The article is organized as follows. In Sec. 2 we overview the relevant experimental work reported in the past two years. Recent theoretical studies of the unique features of intense few-cycle pulses and few-cycle solitons are overviewed in Sec. 3. Finally, Section 4 concludes this paper.

## 2. RECENT EXPERIMENTAL ACTIVITY IN THE AREA OF FEW-CYCLE NONLINEAR OPTICS

In a recent work by Mourou *et al.* [14] a new compression scheme, called *thin film compressor technique*, was advanced, which has the potential to compress a high energy pulse of a few hundred Joules into a single-cycle pulse at the wavelength 800 nm with a focused peak intensity of  $10^{24}$  W/cm<sup>2</sup>; this achievement opens the door to zeptosecond-exawatt extreme nonlinear optics. In fact, as shown in Ref. [14], the thin film compressor technique is able to compress a 25 fs pulse with energy as high as 1 kJ to a pulse with duration of only 1-2 fs. It is expected that using pulses from a high peak power facility generating ultrashort pulses of top-hat like shapes with powers in the range 1-10 PW, combined with the new thin

film compressor technique, one can produce highly focused pulses (the so-called  $\lambda^3$  pulses) of only one femtosecond duration and with peak powers of 100 PW – see Ref. [14] for more details. Here, it is relevant to note a recent brief overview of different types of experiments in *nuclear photonics* and related areas by using the planned Extreme Light Infrastructure - Nuclear Physics (ELI-NP) facility in Bucharest, Magurele, Romania [15]. It is also worth mentioning that in 2013 Chu *et al.* [16] reported on a 2 PW femtosecond laser system at 800 nm, based on the chirped pulse amplification technique using Ti:sapphire crystals; to the best of our knowledge, this 2 PW level is the highest peak power ever achieved from a femtosecond laser system. The compressed pulse width was measured to be 26 fs, and the measured transmission efficiency of the compressor was about 72%, indicating an output energy of about 52.3 Joules for a compressed pulse, which corresponds to a peak power of 2 PW – see Ref. [16]. It can be noted that in order to reach the 10 PW power level, a laser system should be able to produce, for example, 300 Joules of energy in 30 fs pulse durations.

Extreme nonlinear optics is also called *carrier-wave nonlinear optics*; see an excellent introductory book in this area [6]. In a recent comprehensive review article [17], computer simulation techniques for extreme nonlinear optics were reviewed; the emphasis was on the high light-intensity regimes in which both bound and freed electronic states contribute to the medium response and thus affect the optical pulse dynamics.

It is well known that in order to exploit the full potential of single-cycle optical pulses in any application, it is important to obtain the full information about its electric field. Nomura *et al.* [18] advanced a novel pulse characterization scheme, which enables the determination of not only the intensity and phase profiles of the ultrashort pulses, but also their absolute carrier-envelope phase values. The proposed method is based on a combination of frequency-resolved optical gating and electro-optic sampling, and can be used experimentally for a complete characterization of sub-single-cycle infrared pulses [18].

In the area of attosecond physics and applications a rapid progress has been made in the past decade after the first demonstration of isolated attosecond pulses by *high-order harmonic generation* in 2001 [19]. In principle, it is possible to generate few- or even single-cycle optical pulses in the extreme-ultraviolet and soft X-ray spectral domains. Such ultrashort (few- or even single-cycle) optical pulses are currently possible to be generated, in the regimes of visible and infrared wavelengths, by employing the methods of chirped pulse amplification and optical parametric amplification. In 2012, Zhao *et al.* [20] reported the generation of light pulses as short as 67 attoseconds, in the extreme ultraviolet spectral region, with energies of about 90 eV. Earlier relevant works reported generation of such attosecond pulses of lower energies, at 35 eV and 80 eV, see Ref. [21] and Ref. [22], respectively. The production of giant half-cycle attosecond pulses has been reported in 2012 by Wu and Meyer-ter-Vehn [23]. It was thus shown that single

half-cycle pulses with duration of 50 as and high electric fields up to  $10^{13}$  V m<sup>-1</sup> can be produced when irradiating a double foil target with intense few-cycle laser pulses, see Ref. [23] for details of this relevant work. The generation, characterization, and applications of broadband isolated attosecond pulses with spectral bandwidths comparable to the central frequency have been recently reviewed by Chini *et al.* [24]; in the same work, the use of broadband isolated attosecond pulses to the study of bound electron dynamics and to the measurement and control of correlated systems were also discussed.

Whalen *et al.* [25] have predicted a paradigm shift in nonlinear optics associated with ultrashort long-wavelength pulse propagation in gaseous and condensed media. In particular, it was shown that optical carrier shock formation, which modifies the underlying optical waveform, emerges prior to the onset of the self-focusing collapse singularity. The nonlinear partial differential equation adequately describing pulse propagation in this regime, where all nonlinear envelope models fail, was identified as the full field carrier resolved modified Kadomtsev-Petviashvili (MKP) equation. Thus, it was shown, through a series of numerical experiments, that the MKP model can accurately predict carrier wave shock formation for mid infrared and long-wavelength infrared regimes in xenon gas. In the simulations, the input laser beam had a Gaussian spatiotemporal profile with an initial beam waist of 2 mm, the central wavelength in the mid infrared range was 4  $\mu$ m, the pulse duration was 40 fs, whereas the central wavelength for the long-wavelength infrared range was 8  $\mu$ m.

Zhou *et al.* [26] have shown that in strongly phase-mismatched nonlinear frequency conversion crystals, *e.g.*, in bulk lithium niobate (LN) crystals, the pump pulse can experience a large and extremely broadband self-defocusing cascading Kerr-like nonlinearity. If this self-defocusing nonlinearity becomes strong enough to compete with the inherent material self-focusing Kerr nonlinearity, excitation of temporal solitons in the normal dispersion region is possible. Thus, few-cycle soliton compression with noncritical cascaded second-harmonic generation (SHG) was demonstrated. Input energetic 47 fs near-infrared pulses at wavelength 1300 nm with 59 nm FWHM bandwidth and about 200  $\mu$ J pulse energy were compressed in a just 1-mm long bulk LN crystal to 16.5 fs FWHM pulses (below four optical cycles) with 80% efficiency, *i.e.*, with 80% of the initial pulse energy. Also, upon further propagation an octave-spanning *supercontinuum* (SC) was observed. To the best of our knowledge this was the first experimental observation of few-cycle self-defocusing soliton pulse compression in noncritical cascaded second-harmonic generation, which occurred in a short bulk lithium niobate crystal. In a follow-up work [27], the formation and interaction of few-cycle solitons in a LN ridge waveguide were investigated by using numerical techniques. The solitons were created through a cascaded phase-mismatched SHG process, which induces a dominant self-defocusing Kerr-like nonlinearity on the pump pulse. This self-defocusing Kerr-like nonlinearity was combined with normal dispersion in order to

excite such solitons. In this setting, the inherent material self-focusing Kerr nonlinearity is overcome over a wide wavelength range, and temporal solitons are supported from 1100 to 1900 nm, covering the whole communication band. Single cycle self-compressed solitons and SC generation spanning 1.3 octaves were observed when pumped with femtosecond nanojoule pulses at the wavelength 1550 nm. It is worth mentioning that the waveguide was not periodically poled, as quasi-phase-matching would lead to detrimental nonlinear effects impeding the ultrashort (few-cycle) temporal soliton formation. Thus single-cycle pulses were obtained by pumping a low-energy pulse at 1550 nm (50 fs FWHM pulse at 200 kW, *i.e.*, the pulse energy is 10 nJ), whereas by launching a longer pulse with more energy (150 fs FWHM pulses with 30 nJ energy) a SC spectrum covers 1.3 octave bandwidth (1300–3200 nm), see Ref. [27] for details.

It is well known that the generation of coherent ultrashort light pulses with broad spectral bandwidths is of much importance for applications in optical frequency metrology, frequency synthesis, and optical spectroscopy in the mid-infrared spectral range. Such broadband light sources are of particular interest for high-precision frequency comb spectroscopy, see, *e.g.*, the review article [28]. Very recently, Chaitanya Kumar *et al.* [29] reported a few-cycles, broadband, singly-resonant optical parametric oscillator for the mid-infrared spectral domain, based on MgO-doped periodically-poled LN, synchronously pumped by a 20-fs Kerr-lens-mode-locked Ti:sapphire laser. Near Fourier-transform-limited, few-cycle idler pulses tunable in the interval 2179–3732 nm have been generated, with as few as 3.7 optical cycles at the wavelength  $\lambda = 2682$  nm. Note that at the central wavelength of 2682 nm, the measured FWHM idler bandwidth of about 245 nm resulted in a time-bandwidth product of about 0.340, which is very close to the Fourier transform limit of 0.315 corresponding to an ideal  $\text{sech}^2$  pulse. The optical parametric oscillator was continuously tuned over the above mentioned mid-infrared frequencies by cavity delay tuning, providing up to 33 mW of output power at the idler wavelength of 3723 nm, corresponding to a signal wavelength of 1003 nm; see Ref. [29]. It is worth mentioning that by using a different experimental configuration based on MgO-doped periodically-poled LN, broadband five-cycle (50 fs) optical pulses at the central wavelength  $\lambda = 3120$  nm were previously reported by another research group [30].

Ultrashort *optical-vortex pulses* in the few-cycle regime were also investigated in the past few years [31, 32]. In Ref. [31], few-cycle high-contrast vortex pulses with durations around 8 fs (*i.e.*, sub four-cycle vortex pulses) were generated from a Ti:sapphire laser oscillator with a single diffractive-refractive component. The angular and temporal pulse properties were characterized with an advanced time-wavefront sensor. This sensitive wavefront sensor approach might be used for the detection of twisted light (*i.e.* light carrying orbital angular momentum) from rotating black holes and other astronomical objects, see Ref. [31]. Independently, another research group [32] has generated a 2.3-cycle, 5.9-fs,

56- $\mu\text{J}$  ultrashort optical-vortex pulse (ranging from 650 to 950 nm) in the few-cycle regime, by optical parametric amplification. Such ultrashort optical-vortex pulses, of only two cycle duration, can be powerful tools for ultrabroadband and/or ultrafast spectroscopy and for experiments of high-intensity field physics and extreme nonlinear optics [32]. Optical vortices can be also generated in the extreme ultraviolet spectral region using high-harmonic generation [33]; this approach opens the way for several applications in the area of strong-field physics and extreme nonlinear optics, which are based on light beams carrying a screw-type phase distribution [33].

In a recent work [34] it was demonstrated the compression of 35 fs pulses down to a duration of sub-4 fs in a single femtosecond filament; the energy of the observed pulses exceeded 30  $\mu\text{J}$ . This very short pulse duration corresponds to sub-1.5 cycles of the electric field at a carrier wavelength of 780 nm. The generation of intense few-cycle light pulses with stable carrier-envelope phase at wavelengths around 2000 nm was reported in Ref. [35]. In the same work, a simple and efficient concept for the generation of ultrashort infrared pulses with passively stabilized carrier-envelope phase at 100 kHz repetition rate was proposed and realized experimentally. This way, the central wavelength was tunable between 1600 nm and 2000 nm, with pulse durations between 8.2 and 12.8 fs, corresponding to a sub-two-cycle duration over the whole wavelength range; additionally, pulse energies in the range of 100 nJ were achieved; see Ref. [35] for details.

In what follows, we briefly overview a few recent experimental results in the area of white light generation, *i.e.*, in the field of SC generation; see a few excellent reviews in this fast growing research field [36]–[38]. In the process of SC generation the laser light is converted to light with low temporal coherence, while the spatial coherence remains very high. The converted light has a very broad spectral bandwidth spanning even a few octaves in frequency. It is well known [38] that when ultrashort, femtosecond light pulses are launched into highly nonlinear optical fibers with large enough input power (*i.e.*, when the soliton number is  $N > 10$ ), their output spectra become extremely broadened, extending over more than one octave in frequency. Such fiber-based SC sources are presently used in diverse applications, such as biomedical imaging, light spectroscopy (*i.e.*, pump-probe spectroscopy, near-field optical microscopy, coherent Raman spectroscopy, etc.), and frequency metrology. A frequency comb, which is composed of a large number of equally spaced spectral lines, acts as a precise ruler for measurement of optical frequencies. Phase-locked white-light SC pulses came into play and a universal optical-frequency-comb synthesizer was put forward by Bellini and Hänsch [39]; such SC pulses can be used to realize a broad frequency comb for absolute optical frequency measurements from the infrared to the ultraviolet, see Ref. [39].

Multi-octave SC generation in various optical media have been reported in recent years. Specifically, white light generation over three octaves by a femtosecond filament produced in argon with a mid-infrared high-power laser

source has been observed [40]. A broadband SC spanning the spectral range from 350 nm to 5000 nm was observed and about 4% of the input mid-infrared pulses (0.1 TW peak power, 80 fs, 3900 nm) is converted into the 350 nm–1700 nm spectral domain, see Ref. [40]. A multi-octave SC generation from filamentation of mid-infrared femtosecond pulses in a bulk crystal (a 2 mm YAG plate) was reported by Silva *et al.* [41]. The measured spectrum spanned the interval 450 nm–4500 nm, corresponding to 3.3 octaves, with a high spectral energy density of  $2 \text{ pJ nm}^{-1}$ – $10 \text{ nJ nm}^{-1}$ , see Ref. [41]. SC pulse shaping in the few-cycle regime was investigated by Hagemann *et al.* [42]. In the same work, a spectral broadening of the 35 fs input pulses to SC bandwidths was achieved in a controlled two-stage sequential filamentation in air at atmospheric pressure; furthermore, a homogeneous power density over the full spectral envelope in the visible to near infrared spectral range was obtained – see Ref. [42] for details.

In a recent work, Tani *et al.* [43] have investigated PHz-wide supercontinua of nondispersing subcycle pulses (0.5 to 2 fs) generated by extreme modulation instability (MI) of 500 fs pulses (5  $\mu\text{J}$  energy), propagating in gas-filled hollow-core kagome-style photonic crystal fibers. The extreme MI process was used to experimentally generate a high energy (more than 1  $\mu\text{J}$ ) and spectrally broad SC extending from the deep ultraviolet (320 nm) to the near infrared (1300 nm) [43].

SC generation with *optical vortices* (*i.e.*, optical beams carrying a spiral phase) was also reported [44]. In particular, Neshev *et al.* [44] employed an optical vortex beam for the generation of femtosecond SC in a 5 mm thick sample of  $\text{CaF}_2$ . In this solid state nonlinear medium, the spectral broadening and the generation of broad SC were induced by the self-focusing and four-wave mixing processes. Note that the SC generation process was initiated by the filamentation of the optical vortex, resulting in a spatially divergent continuum, see Ref. [44] for more details of this study.

### 3. RECENT THEORETICAL ACTIVITY IN THE AREA OF INTENSE FEW-CYCLE PULSES AND FEW-CYCLE SOLITONS

The continuing experimental progress over the past decade in the study of the dynamics of few-cycle pulses in various types of nonlinear optical media has paved the way for the development of new theoretical approaches to model their propagation in relevant physical settings. It is worth noting that three main classes of dynamical models for FCPs have been put forward in the literature: (*i*) the quantum approaches, (*ii*) the refinements within the framework of slowly-varying envelope approximation (SVEA) of the nonlinear Schrödinger-type envelope equations, and (*iii*) the non-SVEA dynamical models; for a few recent overviews of these main theoretical approaches, see Refs. [45]–[48]. Some of the above mentioned models have been implemented during the past years in the study of

ultrashort *spatiotemporal optical solitons* [49], *i.e.*, ultrashort light bullets, see the recent overview papers [50]–[51]. Here we also draw the reader’s attention to some earlier seminal theoretical studies in the area of nonlinear optics of ultrashort pulses under conditions in which the method of SVEA breaks down, see Refs. [52]–[55].

Drozdov *et al.* [56] have analyzed in detail the unique nonlinear effects associated with the spatiotemporal propagation of few-cycle optical pulses in nonlinear dispersive media, including nonlinearity-induced self-phase modulation, generation of higher harmonics, and the effects of diffraction. In Ref. [56], the effect of the beam diffraction on self-action of Gaussian few-cycle pulses was studied both analytically and numerically. The effect of a subcycle pulse (containing less than a single period of electric field oscillations) on the two-level medium was analyzed numerically by Novitsky [57], beyond the rotating-wave and slowly varying-envelope approximations. For such ultrashort pulses, the breakdown of the area theorem occurs for pulses of large enough area [57]. The same author, using numerical simulations, studied propagation of ultrashort light pulses in an inhomogeneously broadened two-level medium [58]. The self-induced transparency soliton formation and interaction between such solitons in the two-level medium with inhomogeneous broadening of the resonant line were considered in detail in Ref. [58] without the use of the SVEA. The influence of inhomogeneous broadening on the inelastic collisions of counter-propagating solitons in the two-level medium was studied, and it was shown that the level of inhomogeneous broadening has a substantial effect on the elasticity of such collisions [58].

A series of recent theoretical investigations of the so-called *short-pulse equation* (SPE) introduced in Ref. [59] were reported in the past few years. This nonlinear partial differential equation is completely integrable [60], and its exact nonsingular solitary wave solutions can be derived from the breather solutions of the sine-Gordon equation, see Ref. [61]. As shown in Refs. [59] and [62], the results obtained in the framework of the SPE model compare favorably to ones pertaining to the original Maxwell’s equations. Tsitsas *et al.* [63] derived, for the first time to our knowledge, the SPE in the context of nonlinear left-handed metamaterials. In particular, assuming that nonlinear metamaterials are characterized by a weak Kerr-type nonlinearity in their dielectric response, two SPEs were derived for the high- and low-frequency band gaps (where propagation of linear electromagnetic waves is not allowed), with linear effective permittivity  $\epsilon < 0$  and permeability  $\mu > 0$ . In the same work [63], approximate peakon-like and breather-like solitary waves, which can be regarded as weak ultrashort gap solitons, were also presented. Shen *et al.* [64] considered both localized and extended waveforms that arise in the context of the SPE. The exact solutions of the SPE were extracted from the direct analogy of this equation with the integrable sine-Gordon equation. Moreover, Shen *et al.* [64] found by direct numerical simulations that the most robust solution is the breather-type one, although some of the single-

hump variants of the periodic solutions may be preserved upon the time dynamics as well. Multi-peakon, as well as multi-breather and multi-hump waveforms were found to be subject to symmetry breaking instabilities, thus, being less robust on propagation; see Ref. [64] for more details of this theoretical study. Two versions of the SPE in (2+1)-dimensions were derived and studied in detail in Ref. [65]. The relevant models were obtained by using Maxwell's equations and suitable Kramers-Kronig formulas for the permittivity and permeability of the medium, which are relevant to left-handed metamaterials and dielectric slab wave guides, and employing a powerful multiple scales technique. It was shown that ultrashort one-dimensional (1D) breathers appear to be fairly robust in the two-dimensional (2D) setting, while rather general 2D localized initial conditions are transformed into quasi-1D dispersing waveforms [65].

In addition, extended versions of the SPE incorporating higher-order dispersive and nonlinear effects were derived as approximations to the nonlinear wave equation stemming from Maxwell's equations [66]–[67]. The new nonlinear partial differential equations were investigated numerically and it was shown that they capture efficiently such higher-order effects on pulse propagation in cubic (Kerr-type) nonlinear media. The results of Ref. [67] were illustrated by using one- and two-soliton solutions of the first-order short-pulse equation as initial conditions in the nonlinear wave equation. In another recent work, Shen *et al.* [68] revisited the so-called regularized SPE introduced in Ref. [66], and explored the traveling wave solutions of this model. By developing a multiple scale reduction of the regularized SPE to the nonlinear Schrödinger equation, both standing and traveling envelope wave breather type solutions of the former equation, based on the solitary wave structures of the latter equation, were constructed. Both the regular and the breathing traveling wave solutions identified by Shen *et al.* [68] were found to be quite robust in numerical simulations; these results suggest that the experimental observation of such localized structures in the form of few optical cycle solitons becomes possible in a suitable physical setting.

In Ref. [69] the Lie symmetry analysis and generalized symmetry method were performed for a specific short-pulse model. The symmetries for that specific SPE were given, and the phase portraits of the traveling wave solutions were analyzed using the bifurcation theory of dynamical systems. The exact parametric forms of four types of traveling wave solutions were obtained in Ref. [69]: (a) smooth periodic wave solutions, (b) periodic cuspons, (c) loop-soliton solutions, and (d) periodic loop solutions.

Recently, a perturbation theory for the SPE was developed to investigate the effects of various perturbations on ultrashort optical solitons propagating in nonlinear media in the few femtosecond to subfemtosecond (*i.e.*, attosecond) regime [70]. The theory developed in Ref. [70] was formulated using a variational approach since linearization of the exact solution is not tractable. A variety of physically realizable perturbations were considered and the analytic results

presented in Ref. [70] were in agreement with full numerical simulations of the SPE. The propagation of ultrashort solitons in a cubic nonlinear medium modeled by nonlinear Maxwell's equations with stochastic variations of media was studied in a recent work [71]. Three relevant cases were considered: variations of (a) the dispersion, (b) the phase velocity, and (c) the nonlinear coefficient. New stochastic generalizations of the SPE that approximate the solutions of stochastic nonlinear Maxwell's equations were derived by using a modified multi-scale expansion for stochastic systems. Numerical simulations reported in Ref. [71] show that soliton solutions of the SPE propagate stably in stochastic nonlinear Maxwell's equations and that the generalized stochastic SPEs approximate the solutions to the stochastic Maxwell's equations over the distances under consideration.

The dynamics of ultrashort pulses propagating in chiral *carbon nanotubes* (CNTs) – *i.e.*, nanotubes with helical symmetry – was investigated by Konobeeva and Belonenko [72]. The corresponding wave equation for the electromagnetic field propagating in chiral CNTs has been analyzed and a phenomenological equation for the vector potential, similar to the integrable sine-Gordon equation, has been derived and its numerical solution has been obtained; see Ref. [72] for details of this theoretical study. In a follow-up study performed by the same group [73], the unique dynamics of ultrashort electromagnetic pulses in chiral CNT waveguides in the presence of external alternating electric fields was studied by adequate numerical techniques. Zhukov *et al.* [74] investigated the propagation of extremely short electromagnetic three-dimensional bipolar pulses in arrays of semiconductor CNTs. In Ref. [74], the heterogeneity of the pulse field along the axis of the CNTs was accounted for the first time, to the best of our knowledge. The evolution of the electromagnetic field and the charge density of the sample were described by Maxwell's equations supplemented by the continuity equation. Thus, both electromagnetic field intensity distribution and electron density distribution at different times were analyzed. The rather complex study performed by Zhukov *et al.* [74] revealed the possibility of propagation of three-dimensional electromagnetic breathers in CNT arrays. In another recent work by Zhukov *et al.* [75], the propagation of ultrashort electromagnetic pulses through CNT arrays with multi-level impurities has been investigated and the effects of the hopping integrals and band gap of deep impurities on the pulse tail decay have been uncovered.

Kozlov and Rosanov [76] have performed comprehensive numerical simulations of few-cycle pulses generated by a laser in the regime of *coherent mode locking*. It was shown that it is possible the generation of ultrashort pulses with duration close to the reciprocal of the main laser transition frequency of the gain medium for a wide range of relevant laser parameters. In Ref. [77], by the same research group, the generation of single-cycle pulses from a passively-mode-locked laser (*i.e.*, the laser is operating in the regime of coherent mode locking) was theoretically demonstrated for the case when the resonant lines of both the gain and absorbing media are inhomogeneously broadened. In this specific

physical setting, in contrast to conventional mode-locked lasers, it was shown in Ref. [77] that the stable single-cycle pulse operation persists even when the spectral widths of the inhomogeneously broadened lines of the amplifier and the absorber become nearly as wide as the central resonance frequency of their two-level transitions.

Xiao *et al.* [78] have applied the time-transformation method for studying the propagation of few-cycle optical pulses inside a nonlinear Kerr medium after taking into account that changes in the refractive index vary with the electric field as  $E^2$  and not by its average over an optical cycle. Thus, the powerful time-transformation technique was employed in Ref. [78] to study the propagation of few-cycle pulses in a nonlinear cubic medium with the time-dependent refractive index of the form:  $n = n_0 + n_2 E^2(t)$ . This numerical method correctly predicts carrier-wave shocking and generation of odd-order harmonics inside a Kerr (cubic) nonlinear medium. Xiao *et al.* [78] extended their technique to study the impact of a finite response of the Kerr nonlinearity on harmonic generation and to include chromatic dispersion that cannot be ignored for ultrashort (few-cycle) pulses. The time-transformation method provides an alternative to the well-known finite-difference time-domain numerical technique, as it deals with the electric field directly but does not require typical step sizes to be a small fraction of the wavelength. As a result, the computation speed was greatly enhanced; see Ref. [78] for a detailed study of these issues.

Amiranashvili *et al.* [79] have studied the propagation of few-cycle optical solitons in nonlinear media with an anomalous, but otherwise arbitrary, dispersion and a cubic (Kerr-type) optical nonlinearity, beyond the slowly-varying envelope approximation. The typical parameters for bulk fused silica at two carrier wavelengths (1600 nm and 2500 nm) were used in the numerical simulations performed in Ref. [79]. The optical field was derived directly from Maxwell's equations under the assumption that generation of the third harmonic is a nonresonant process or, at least, cannot destroy the pulse prior to inevitable linear damping. Thus, neither the slowly-varying envelope approximation nor the directional approximation was used in Ref. [79]. In fact, Amiranashvili *et al.* [79] obtained the solitary wave solutions by adequate numerical techniques up to nearly single-cycle duration using the *spectral renormalization method* originally developed for the envelope optical solitons containing a large number of optical oscillations (*i.e.*, many optical cycles). The analysis performed in the same work has revealed that for ultrashort few-cycle pulses the main contribution comes from the higher-order linear dispersion, and to a lesser extent, from the self-steepening term. The few-cycle solitons are finally destroyed due the unavoidable Cherenkov radiation; see Ref. [79]. In a follow-up paper, Amiranashvili *et al.* [80] have considered the propagation of ultrashort optical pulses in nonlinear fibers and have suggested a new theoretical framework for the description of pulse dynamics and exact characterization of solitary solutions. Their quite interesting and powerful

approach deals with a proper complex generalization of the nonlinear Maxwell equations. In their approach the use of the slowly-varying envelope approximation was completely avoided. The only essential restriction in the approach developed in that work is that fiber dispersion does not favor both the so-called Cherenkov radiation and the resonant generation of the third harmonics; these two effects destroy ultrashort (few-cycle long) optical solitons. In the case of arbitrary anomalous dispersion and cubic (Kerr-type) nonlinearity, a continuous family of solitary wave solutions connecting fundamental solitons to nearly single-cycle ultrashort ones was derived; see Ref. [80].

In a recent work [81], new breather-type solutions of the integrable modified Korteweg-de Vries–sine-Gordon equation describing few-optical-cycle solitons beyond the slowly varying envelope approximation were reported. It is well known [47] that the modified Korteweg-de Vries–sine-Gordon equation provides a general (1+1)-dimensional model describing propagation of ultrashort (few-cycle long) optical solitons in two-component nonlinear media. For this integrable nonlinear model, the soliton solutions in their Wronskian form were given in Ref. [81]. The obtained solutions describe interactions between solitons, breathers and their limit forms; see Ref. [81] for a detailed study of these problems. Xu *et al.* [82] derived from a system of complete Maxwell-Bloch equations, without using the slowly-varying envelope approximations, the rotating reduced Maxwell-Bloch equations, which describe the propagation of few-cycle pulses in a transparent medium with two isotropic polarized electronic field components. Families of rational soliton solutions, including the so-called *ultrashort rogue waves* (*i.e.*, few-cycle optical rogue waves), were constructed explicitly through degenerate Darboux transformation method. Also, the unique evolution dynamics of the first-, second-, and third-order few-cycle optical rogue waves was investigated in detail by Xu *et al.* [82]; for other recent theoretical investigations on optical rogue waves see Refs. [83, 84].

Marini and Biancalana [85] have investigated analytically and numerically the interband self-induced transmission of surface plasmon polaritons in a gold film surrounded by an external Kerr (cubic) nonlinear optical medium. In fact, the phenomenon of *self-induced transparency* was exploited in order to suppress the interband absorption of surface plasmon polaritons. The optical propagation was modeled by using a version of the generalized nonlinear Schrödinger equation for the field envelope, coupled to Bloch equations for valence electrons of gold, predicting self-induced transparency of ultrashort plasmon solitons with pulse duration below 10 fs. It was shown that the Kerr nonlinearity from the surrounding dielectric can be effectively used to compensate for the group velocity dispersion. Thus, a possible experimental verification of these theoretical predictions might be achieved with pulse peak powers of the order of 10 kW and pulse time durations of less than 10 fs.

Porras [86] has described the temporal evolution of the electric field of few-cycle optical pulses with arbitrary, time-varying polarization states by means of the instantaneous polarization ellipse and phase. In particular, in Ref. [86], a physically meaningful definition of carrier-envelope phase for arbitrarily polarized pulses was introduced and it was further used to study the changes in the temporal evolution of the electric field of such ultrashort pulses. During propagation, significant changes in the polarization state, phase, and carrier-envelope phase were reported by Porras [86], for the first time, to our knowledge. Liang Guo *et al.* [87] have performed a comprehensive theoretical investigation of the propagation of few-cycle laser pulses in resonant two-level dense media with a subwavelength structure, which is described by the full Maxwell-Bloch equations without using the slowly varying envelope and rotating wave approximations. It was shown in Ref. [87] that the input laser pulses can be shaped into shorter ones with a single or less than one optical cycle. The results reported in this work show that media with a subwavelength structure can significantly shape few-cycle pulses into a subcycle pulse, even for the case of input chirped laser pulses. Half-optical-cycle damped solitons in quadratic nonlinear media have been also investigated [88] by using the Korteweg-de Vries-Burgers equation without using the SVEA in the long-wave regime, *i.e.*, assuming that the characteristic frequency of the pulse is much lower than the resonant frequency of the atoms; see Ref. [88] for details.

Recently, Babushkin and Bergé [89] have obtained the fundamental solution of a variant of the three-dimensional wave equation known in the literature as *unidirectional pulse propagation equation* (UPPE), which was proposed to deal, *e.g.*, with the propagation of few-cycle pulses through waveguides, or in a form of long filaments of light in weakly ionized, optical transparent media [90]. It was shown in Ref. [89] that the UPPE describes wave propagation in both longitudinal and temporal directions and, thereby, its fundamental solution breaks the causality principle.

Todorov *et al.* [91] have studied the spatiotemporal dynamics of high-intensity femtosecond laser pulses within a rigorous physical model. Parameter values considered in Ref. [91] referred to an initial optical waveform having 0.5 mJ pulse energy, 150 fs FWHM pulse duration, 300 mm of transversal width (beam diameter), and 800 nm central wavelength. The initial pulse has a Gaussian spatiotemporal shape and propagates in argon at a pressure of 18 atm. At properly chosen conditions, self-compression of the pulse and stable propagation of the compressed pulse was found in this work [91]. The stabilization of the pulse results mainly from a balance between competitive nonlinear optical processes. The strong influence of ionization on the group velocity dispersion, leading even to inversion from a positive to a negative value, was found by considering realistic physical conditions; see Ref. [91] for details. In a recent relevant work [92], the carrier-envelope phase dependence of unipolar half-cycle soliton pulse generation when few-cycle pulses propagate in resonant asymmetric (*i.e.*, noncentrosymmetric)

media was studied in detail. It was shown that the generated half-cycle solitons depend on both the asymmetry of the two-level system and the electric field shape of the incident few-cycle pulse. The key role of carrier-envelope phase of the incident pulse on the number and amplitudes of generated half-cycle solitons was uncovered, see Ref. [92].

#### 4. CONCLUSIONS

In summary, in this paper we have attempted to provide the interested reader with a general overview of the current state-of-the-art of the continuously growing research area of nonlinear optics of intense few-cycle pulses in a large variety of physical settings. We have briefly described both experimental and theoretical results that have recently appeared in the literature. The described experimental results mainly referred to studies on few-cycle pulses and vortices in the 1D and 2D dimensional settings, respectively, as well as to supercontinuum generation. On the other hand, the presented theoretical results mainly dealt with the development and investigation of models, as well as their few-cycle pulse and soliton solutions. Importantly, the intense experimental efforts and corresponding impressive results have inspired and triggered the recent theoretical investigations, but also the increasing amount of theoretical work has accompanied and driven the experimental progress.

We conclude with the hope that this brief overview on recent exciting theoretical and experimental developments in the field of nonlinear optics of ultrashort pulses and of few-cycle optical solitons will inspire further studies. Finally, we do believe that interesting times have arrived for this rapidly developing research area, and new exciting results – both in experiments and in theory – will appear soon.

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