Enhancing heat transfer in vortex generator-type multifunctional heat exchangers

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A B S T R A C T

Global and local analysis of the heat transfer in turbulent vortical flows is studied using three-dimensional numerical simulations. Vorticity is generated by inclined vortex generators in a turbulent circular pipe flow with twelve different configurations that fall into three categories. In the first category are rows of trapezoidal vortex generators in different arrangements; in the second category the vortex generators are fixed at certain distance from the tube wall, and the third category has vortex generator rows between which a row of small protrusions are inserted on the tube wall. First, a global analysis of the thermal performance is performed for all these configurations, which are also compared with other heat exchangers from the literature. New correlations for the friction factor and Nusselt number are then obtained. Local analysis of the effect of the flow structure on the temperature distribution is carried out for the four configurations showing the best performances. The local analysis involves studying the streamwise vorticity flux to characterize the convective transport process, the turbulent kinetic energy characterizing the turbulent mixing, and finally the local Nusselt number.

1. Introduction

Artificially generated vorticity is efficient in fluid mixing and heat transfer since it enhances the exchange of fluid particles between the different flow regions with relatively small increases in pressure loss [1]. Several methods exist for generating vorticity, such as Görtler vortices and Dean cells induced by wall curvature and/or rotation [2,3], embedded vortices generated by jets [4], and turbulence promoters or vorticity generators [5,6]. This last category is the main interest in the present study.

Physical analysis of vorticity effects on heat- and mass-transfer mechanisms is a fundamental issue in optimizing existing heat exchangers and devising new enhanced designs. In fact, two types of vorticity can be distinguished: transverse stagnant vortices that are local recirculations behind the vortex generator and have their axis of rotation perpendicular to the main flow direction, and streamwise vortices advecting in the flow direction with a swirling motion [5]. It was shown that most heat- and mass-transfer enhancement is provided essentially by streamwise vorticity, while the transverse stationary vorticity slightly enhances heat transfer in the region near the vortex generator, generally in the near-wake of the vortex generator [1,5]. This fact is demonstrated by the strong relationship between the streamwise vorticity flux and the span-averaged Nusselt number observed downstream from rectangular [7] and trapezoidal [8] vortex generators.

In the present work, pressure-driven longitudinal vorticities are generated in a turbulent flow by using different configurations of vortex generator rows inserted in a circular tube. These vortex generators are based on the trapezoidal mixing tabs used in the high-efficiency vortex (HEV) static mixer [9] (see Fig. 1a). Flow past trapezoidal vortex generators has been extensively studied due to its ability to enhance turbulent mixing, mass transfer and phase dispersion by the generation of complex coherent structures [10–12]. Mainly, two types of flow structures are observed downstream from a trapezoidal vortex generator: a counter-rotating vortex pair formed by the pressure difference across the vortex generator, and a periodic sequence of horseshoe-like structures shed from the trailing edges of the vortex generator [13]. These structures have been shown to enhance fluid mixing and heat exchange between low-momentum near-wall regions and the high-momentum zone in the flow core.

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The efficiency of these vortex generators allows the integration of several unit operations such as mixing, chemical reaction, and heat transfer into a single system. This implies the integration of several unit operations such as mixing, chemical reaction, and heat transfer into a single system. This implies the integration of several unit operations such as mixing, chemical reaction, and heat transfer into a single system. This implies the integration of several unit operations such as mixing, chemical reaction, and heat transfer into a single system. This implies the integration of several unit operations such as mixing, chemical reaction, and heat transfer into a single system. This implies the integration of several unit operations such as mixing, chemical reaction, and heat transfer into a single system. This implies the integration of several unit operations such as mixing, chemical reaction, and heat transfer into a single system. 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where the model constants are $C_m = 0.0845$, $C_{1_\varepsilon} = 1.42$ and $C_{2_\varepsilon} = 1.68$.

The additional term $R_\varepsilon$ on the right-hand side of Eq. (4) is given by [18]:

$$R_\varepsilon = \rho C_m \eta^3 \left( 1 - \frac{\eta}{4.38} \right) \frac{\varepsilon^2}{1 + 0.012 \eta^3} \frac{k}{k}$$

with $\eta = k \varepsilon / \varepsilon$. It should be noted here that the RNG $k-\varepsilon$ model shows fundamental improvements over the standard $k-\varepsilon$ model [19] since the effects on turbulence of strong streamline curvature, vortices and swirl effect are taken in account, thus enhancing the final solution [20].

The heat transfer is computed by solving the energy equation

$$\frac{\partial}{\partial x_i} [\rho C_p (\rho E + p)] = \frac{\partial}{\partial x_j} \left( \lambda_{\text{eff}} \frac{\partial T}{\partial x_j} + u_i \frac{\partial \tilde{T}}{\partial x_j} \right)$$

Fig. 1. (a) Three-dimensional view of the HEV mixer and front view of the (b) aligned (Alg) and (c) alternating (Alt) arrays arrangements; here the black trapezoidal tabs correspond to the arrays $n$ and the gray tabs are the tabs in the array $n + 1$ with $\theta = 45$ the tangential rotation angle of the tab rows. (d) The distribution of the protrusions downstream from the tabs array.
where $E$ is the total energy, $\lambda_{eff}$ the effective thermal conductivity and $\kappa_{\delta}$ the deviatoric stress tensor representing the viscous heating due to shear effects, i.e. due to velocity gradients for incompressible viscous fluid flows.

The solver used for the flow computation is the CFD code Fluent 6.3, which is based on an Eulerian approach to solving the Cauchy equations through cell-centered finite volume discretization. The flow equations are solved sequentially with double precision and a second-order upwind scheme for spatial discretization of the convective terms [21]. The diffusion terms are central-differenced and second-order accurate. Pressure–velocity coupling is achieved by the SIMPLE algorithm [22].

The flow in the near-wall region is solved by a two-layer approach in which the near-wall region is divided into a viscous sublayer and a turbulent region. These two regions are delimited by the turbulent wall Reynolds number based on the distance normal to the wall $y$, which is defined as $Re_w = \frac{yu}{v}$. In the viscous sublayer, for $Re_w < 200$, the one-equation model of Wolfstein [23] is used, where only the momentum and turbulent kinetic energy transport equations are solved (Eqs. (2) and (3)), and the turbulent dissipation rate $v$ is related to the turbulence kinetic energy $k$ by an empirical formula defined by Chen and Patel [24]. This approach requires estimation of the wall-adjacent cell size corresponding to an ideal dimensionless wall distance $y^+$ of no more than 5, ensuring that the viscous sublayer is meshed. Therefore, the size of the cells near the wall region is refined using the tool provided in Fluent 6.3, thus refining the near-wall cells for which $y^+ > 5$.

The conduction in the solid tabs is taken into account by solving the heat conduction equation in a solid $\frac{\partial^2(T_{\text{solid}})}{\partial x^2} = 0$. The material for the vortex generator was chosen as aluminum so that it can be compared with other heat exchangers in the open literature, as shown in section 3.2 below.

The working fluid is water, with physical properties assumed independent of temperature [25]. Flow and heat-transfer simulations are carried out for Reynolds numbers 7500, 10,000, 12,500 and 15,000 and for uniform wall temperature $T_w = 360\, \text{K}$. The velocity profile for fully developed turbulent pipe flow [26] is prescribed at the inlet of all computational domains with uniform temperature $T_{in} = 298\, \text{K}$. The turbulence kinetic energy and its dissipation rate are estimated by the turbulence intensity $I$ derived from the empty-tube equilibrium state given by Hinze [26]. Atmospheric pressure is prescribed at the outlet. No-slip boundary conditions are prescribed on the vortex generator surfaces and on the pipe wall.

The residual value $10^{-6}$ is set as the convergence criterion for the solutions of the flow equations in Eqs. (1–4). Beyond this value no significant changes are observed in the velocity field and turbulence quantities. A value of $10^{-8}$ is set for the convergence criterion of the solution of the energy equation (Eq. (6)).

2.2. Computational domain and meshing

In the present study twelve different flow configurations (HEV static mixer flow geometries) are investigated. The HEV mixer consists of a circular tube of inner diameter $D = 20\, \text{mm}$ in which seven vortex generator arrays are inserted. Each array is composed of four diametrically opposed trapezoidal vortex generators inclined to the wall at an angle $\beta = 30^\circ$. The distance of the upper edge of the vortex generator from the wall is $h = 3.9\, \text{mm}$. The total length of the tube is $L = 140\, \text{mm}$ and the distance between two successive tab rows is $d = D = 20\, \text{mm}$. More details on the tab dimensions are provided in Habchi et al. [27].

The difference among the twelve flow configurations lies in modifications of the vortex generator shape; these are summarized in Table 1, which shows the modifications for a single tab. The tabs can be directly inclined in the main flow direction (Dir) or inversely inclined (Inv) to the main flow direction. The first configuration presented in Table 1a. Alg–Inv is used for numerical validation by comparing the present results for this geometry with those obtained previously with a different numerical model and by laser Doppler velocimetry (LDV) [16]. It should be noticed that two types of arrays arrangements are studied; the first is for aligned arrays (Alg), where the seven tabs rows are aligned, and the second is the alternating arrays (Alt), where the tab arrays undergo a periodic $\theta = 45^\circ$ tangential rotation with respect to one another, as shown in Fig. 1. The vortex generator in configuration (b) Alg–Inv have the same shape as in (a) Alg-Inv, but the arrays are periodically rotated one with respect to another; see Fig. 1c.

The tabs shown in Table 1 (c–f) are supported to be at certain distance $e$ from the tube wall (this distance is shown in gray) with $e = 1\, \text{mm}$ (V1) or $e = 2\, \text{mm}$ (V2). The thickness of the vortex generator legs at their attachment to the wall is 0.25 mm. This distance allows the flow to pass below the vortex generator so as to reduce the recirculation flow region in the near-wake of the vortex generator, and thus to reduce effluent clogging. The tabs presented in Table 1 (g, h) are also supported to be at a distance $e = 1\, \text{mm}$ (D1) from the wall, but the projected surface removed from the bottom part of the tab is added at the top of the vortex generator.

In configurations (i) Alg–Inv–Rev and (j) Alg–Dir–Rev, the trapezoidal vortex generator is reversed where the small base is attached to the wall. The configuration in Table (k) Alg–Shf consists of seven aligned arrays in which the odd arrays are inclined in the opposite direction to the flow (Inv) and the even arrays are inclined in the flow direction (Dir).

In the configuration in Table 1 (m) Alg–Inv–Rev–Prot and Fig. 1d, eight protrusions consisting of hemispheres of radius $r = 1\, \text{mm}$ are uniformly distributed on the inner wall of the tube at a distance $l = 3\, \text{mm}$ downstream from each vortex generator array. In this configuration, the vortex generators are the same as in (i) Alg–Inv–Rev and (j) Alg–Dir–Rev. A nonuniform unstructured three-dimensional mesh with hexahedral volumes is performed using the software Gambit. Special attention is paid to the near-wall refinement at all solid boundaries (wall and tab surfaces) so as to take into account the high velocity and temperature gradients in these regions.

To determine the appropriate mesh density for solution grid independence, the solver is run with increasing mesh densities until no effect on the results is detected. The criterion for the grid sensitivity test is based on the turbulence kinetic energy and on velocity and temperature profiles at the outlet of the computational domain. The mesh refinement is performed for each of the twelve configurations. The mesh is refined in regions in which large gradients are observed in the turbulence kinetic energy, velocity and temperature profiles. The final number of cell ranges between 10M and 17M cells, depending on the flow configuration. The numerical simulations are performed on four parallel processors.

The present work aims to study numerically different flow configurations of a HEV static mixer that mainly consists of seven rows of vortex generator and is generally preceded by a preconditioner to produce a fully developed flow at its inlet [9,11]. Seven vortex generator rows are therefore used here for the twelve configurations with fully developed hydrodynamic conditions at the inlet. This choice may be also useful for future experimental laboratory studies.

3. Results and discussion

3.1. Numerical validation

The first configuration, (a) Alg–Inv, has been studied experimentally by using laser Doppler velocimetry (LDV), and
Table 1
The different tab shapes for the twelve configurations studied: (a) side view and (b) front view of one tab.

(a) Alg–Inv
(b) Alg–Inv

(c) Alg–Inv-V2 (e = 2 mm)
(d) Alg–Inv-V1 (e = 1 mm)

(e) Alg–Dir-V2 (e = 2 mm)
(f) Alg–Dir-V1 (e = 1 mm)

(g) Alg–Inv-D1 (e = 1 mm)

(h) Alg–Dir-D1 (e = 1 mm)

(i) Alg–Inv–Rev

(j) Alg–Dir–Rev

(k) Alg–Shf

(m) Alg–Inv–Rev–Prot (r = 1 mm)
Comparison of Nusselt number $Nu$ and pressure drop $\Delta p$ for different turbulence models.

<table>
<thead>
<tr>
<th></th>
<th>$Nu$</th>
<th>Relative error in $Nu$</th>
<th>$\Delta p$ (Pa)</th>
<th>Relative error in $\Delta p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present results ($k-\varepsilon$ RNG)</td>
<td>389.4</td>
<td>--</td>
<td>1354.2</td>
<td>--</td>
</tr>
<tr>
<td>Habchi et al. [16] ($k-\varepsilon$ Std)</td>
<td>391.5</td>
<td>0.5%</td>
<td>1393.6</td>
<td>2.8%</td>
</tr>
<tr>
<td>Habchi et al. [16] (RSM)</td>
<td>420.0</td>
<td>7.4%</td>
<td>1451.5</td>
<td>6.7%</td>
</tr>
</tbody>
</table>

The global Nusselt numbers presented in Table 2 are obtained from the equation

$$ Nu = \frac{mcp}{\pi L} \frac{T_{b, outlet} - T_{b, inlet}}{T_w - T_{mean}} $$

(7)

where $m$ is the mass flow rate, $L$ the heat exchanger length, $\lambda$ the thermal conductivity of the working fluid (water here), and $T_{b, inlet}$ and $T_{b, outlet}$ are the bulk temperatures respectively at the heat exchanger inlet and outlet, and $T_{mean} = (T_{b, inlet} + T_{b, outlet})/2$. In Table 2, $\Delta p$ is the pressure difference between the outlet and the inlet cross sections of the heat exchanger.

Table 2 suggests that the results from both $k-\varepsilon$ RNG and $k-\varepsilon$ Std models are in good agreement with the results from the RSM model for both Nusselt number and pressure losses. The relative error for the Nusselt number between $k-\varepsilon$ RNG and RSM models is 7.4% and 6.7% for the pressure losses. The relative errors in Table 2 are calculated by: $\left| \frac{(k-\varepsilon$ RNG) - (k-\varepsilon Std)/(k-\varepsilon Std) \right| $ for relative error between Std and RNG $k-\varepsilon$ turbulence models, $\left| (k-\varepsilon$ RNG - (RSM)/(RSM) $\right|$ for relative error between RSM and $k-\varepsilon$ RNG turbulence models.

Meanwhile, the $k-\varepsilon$ RNG model gives results in good agreement with the RSM model while using much less computational time. Therefore, the $k-\varepsilon$ RNG model is used to study the different twelve flow configurations for four different Reynolds numbers.

3.2. Global performance analysis

In this section a global analysis of the performance of different configurations is presented. The global Nusselt number obtained from Eq. (7) is presented in Fig. 3 for all the geometries and for the four Reynolds numbers ranging between 7500 and 15,000.

For single-phase flows, the Nusselt number can be correlated with the Reynolds number via a power-law [14, 15]:

$$ Nu = aRe^bPr^{0.4} $$

(8)

where $Pr = \mu c_p/\lambda$ is the Prandtl number for the working fluid.

It is worth noting that higher coefficient $b$ means a faster increase in $Nu$ with $Re$ and thus better heat transfer augmentation with mean flow velocity increase. Generally, for heat exchangers based on circular pipe with inserts or vortex generators, the coefficient $b$ in Eq. (8) ranges between 0.3 and 0.8 [14, 15]. The Nusselt number correlations for the present configurations in Table 3 show that the coefficient $b$ ranges between 0.6 and 0.85. The application
range for these correlations corresponds to Reynolds number between 7500 and 15,000. These Reynolds numbers correspond to those used in industrial applications for this type of multifunctional heat exchangers/reactors.

In both Fig. 3 and Table 3, it can be noticed that Alg–Inv–V2 (c) and Alg–Dir–V2 (e) have the lowest Nusselt numbers. In fact, in these two configurations, the distance from the wall of the vortex generator is 2 mm from the wall. This distance reduces the contact surface between the heated wall and the vortex generator and thus reduces the conduction heat flux through the vortex generator tab; as a result, convective heat transfer between the vortex generator and the surrounding fluid is decreased. Meanwhile, the configurations Alg–Inv–D1 (g) and Alg–Dir–D1 (h), in which wet surfaces are kept equal to those in the reference geometry Alg–Inv (a), have higher Nusselt numbers than Alg–Inv–V1 (d) and Alg–Dir–V1 (f). However, the pressure losses are greater, thus reducing the global heat transfer performance of the heat exchanger, as discussed in the next section. The highest Nusselt numbers are obtained for the configurations Alg–Inv (a), Alg–Inv–Rev (i), Alg–Dir–Rev (j) and Alg–Inv–Rev–Prot (m). Of them all, the Alg–Inv–Rev–Prot (m) geometry gives slightly higher Nusselt number when the Reynolds number exceeds 12,500, with an increase of about 10% relative to the Alg–Inv–Prot (a) reference configuration.

The pressure losses $\Delta p$ can be represented by the friction factor $f$ as:

$$f = \frac{2D}{U_0} \frac{\Delta p}{\rho U_0^2}$$

with $U_0$ the mean flow velocity, $\Delta p$ the computed pressure drop between the heat exchanger inlet and outlet, and $\rho$ the working fluid density.

In turbulent flows, the global friction factor $f$ is correlated to the Reynolds number as [14,15]:

$$f = aRe^b$$

The correlations for $f$ in the configurations studied here are presented in Table 3, and $f$ is plotted versus $Re$ in Fig. 4. It can be observed from this figure that the pressure losses decrease with increasing the distance of the vortex generator from the wall due to the decrease in the projected wet surface. The highest pressure losses are observed for the configurations Alg–Inv (a), Alg–Inv–Rev (i), Alg–Dir–Rev (j) and Alg–Inv–Rev–Prot (m), in which the highest Nusselt numbers were also observed.

The separate analyses of the Nusselt number and the friction factor do not give a clear idea of the global performance of the different geometries. Therefore, to compare the heat-transfer efficiency of the different configurations for constant pumping power, a thermal enhancement factor $\eta$ is introduced, defined as the ratio of the HEV convective heat transfer $H$ to that in a straight-pipe flow $H_0$ [30,31]:

$$\eta = \left( \frac{Nu}{Nu_0} \right)^{1/3}$$

where $Nu_0$ is the Nusselt number for a turbulent straight-pipe flow. Kakac et al. [32] concluded that the Gnielinski [33] correlation (Eq. (12) below) agrees with the available data better than any other expression over a range of Prandtl numbers, $Pr$, from 0.5 to 200 and Reynolds numbers from 2300 to $5 \times 10^6$ for fully developed turbulent flow in circular tubes:

$$Nu_0 = \left( \frac{f_0}{8} \right)^{1/3} \frac{Pr}{1 + 12.7(f_0/8)^{1/2} (Pr^{2/3} - 1)}$$

where $f_0$ is the friction factor in straight-pipe turbulent flow following the Blasius formula:

$$f_0 = 0.079 Re^{-0.25}$$

Fig. 5 presents the thermal enhancement factor $\eta$ versus Reynolds number for the different flow configurations. An
interesting observation is that, beyond $Re = 10,000$, the thermal performance is increased in the Alg−Rev−Inv−Prot (m) configuration relative to the Alg−Inv (a) reference configuration, reaching about 10% for $Re = 15,000$. Moreover, the slope of the power-law fitting for the Alg−Rev−Inv−Prot (m) configuration is smaller than those for the other geometries, implying that the decrease in the thermal enhancement factor $\eta$ with Reynolds number is moderate and thus it is more efficient.

The poorest performance is observed in the Alg−Inv (b) configuration, since the periodic alternating arrays arrangement (Fig. 1 (c)) increases greatly the pressure losses with no significant increase in the Nusselt number. Note that the values of all thermal enhancement factors $\eta$ range between 1.7 and 2.8, meaning that the present configurations enhance the heat transfer performance between 70% and 180% relative to that of an empty pipe. These values of $\eta$ are higher than those obtained in the literature for different shapes of ribs [30] and twisted tape inserts [31], whose thermal enhancement factors range between 0.5 and 1.5.

Fig. 6 shows the global Nusselt number versus the global power dissipation rate $\tau$ per fluid unit mass (W/kg), which is related to the pressure drop $\Delta p$ by:

$$\tau = \frac{U_m \Delta p}{\rho L} \quad (14)$$

The present configurations are compared in Fig. 6 to other heat exchangers from the literature [30,31,34–39] for Reynolds numbers ranging from 7500 to 15,000. In this figure, the straight-pipe turbulent flow presents the lowest performance since it lies in the region of lowest Nusselt number and power dissipation. The configurations studied here, which are grouped in the region $3 < \tau < 8$, show better performances than the other heat exchangers using ribs and helical inserts. In fact, the helical Kenics [39] has almost the same performance as the Alg−Dir configuration, which is the classical configuration for the HEV static mixer [9,25,28] (as shown in the zoomed inset graph). However, the performance intensification is improved in all the present configurations, especially in Alg−Inv−Rev−Prot (m), with only a small increase in the power consumption.

Fig. 6 enlarges the region $0.5 < \tau < 10$ and $150 < Nu < 450$ to show the most efficient configurations. It is observed that the slopes of $Nu$ versus $\tau$ differ from one configuration to another. In fact, higher slope implies that greater augmentation in $Nu$ can be provided with a smaller increase in power dissipation rate $\tau$. Comparing both Alg−Inv (a) and Alg−Inv−Rev−Prot (m) shows that the slope of the latter is 30% higher than that of the former and that it thus provides better performance. Moreover, relative to the classic HEV geometry used in the industry [9], here called Alg−Dir, the Alg−Inv−Rev−Prot (m) configuration shows higher heat-transfer efficiencies, ranging between 40% and 55% with an increase in power consumption ranging between 40% and 45%.

In the next section, the four configurations presenting the best heat transfer performance are chosen for detailed local analysis. These configurations are Alg−Inv (a), Alg−Inv−V1 (d), Alg−Inv−Rev (i) and Alg−Inv−Rev−Prot (m).

### 3.3. Local analysis of the flow structure and heat transfer

Fig. 7 shows the temperature distribution and velocity vectors for Reynolds number 15000 at the HEV cross section just downstream from the 7th tabs array, i.e. at the outlet cross section of the heat exchanger. A counter-rotating vortex pair is observed in each configuration that is caused by the pressure difference between the wake of the vortex generator (region with low-momentum flow) and the flow core including high-momentum flow. This counter-rotating vortex pair plays the role of internal agitator and its effect is clearly observed on the radial temperature distribution. The common inflow in the symmetry plane of the counter-rotating vortex pair, i.e. the radial flow from the flow core towards the wall, reduces the thermal boundary-layer thickness near the wall, thus increasing the temperature gradient and the heat-transfer coefficient. Conversely, the common outflow between two neighboring counter-rotating vortices ejects high-temperature fluid particles from the near-wall region towards the flow core, thus enhancing the fluid mixing process. No significant differences are observed in Fig. 7 among the flow topology of the counter-rotating vortex pair in the different geometries; however, the temperature distribution...
differs greatly among the configurations because of the difference in the intensity of the counter-rotating vortex pairs. This intensity can be quantified by the vorticity flux $J_z$ which is obtained by integrating the absolute streamwise vorticity $|\omega_z|$ on the flow cross section $S$ [7,8]:

$$J_z = \frac{\sum_{i=1}^{n} |\omega_z| dS}{\sum_{i=1}^{n} dS}$$

where $n$ is the number of cell faces in the given cross section $S$.

The longitudinal variation of the vorticity flux $J_z$ is presented in Fig. 8 for different cross sections in the last array of vortex generators. Three configurations, Alg–Inv (a), Alg–Inv–Rev (i) and Alg–Inv–Rev–Prot (m), present similar $J_z$ variations with larger values than the Alg–Inv–V1 (d) configuration. In these three geometries, $J_z$ starts increasing from the leading edge of the vortex generator, where the formation of the counter-rotating vortex pair occurs, and reaches its maximum at the trailing edge of the vortex generator. Downstream from the vortex generator, the vorticity flux $J_z$ decreases due to viscous effects. At the position of the protrusion in the Alg–Inv–Rev–Prot (m) configuration, a slight increase in $J_z$ is observed relative to the other two geometries, due to the local (though weak) vorticity generated by the protrusion; this is shown in Fig. 9, which focuses on the protrusion in the temperature distribution and streamlines in the Alg–Inv–Rev–Prot (m) configuration.

In fact, the streamwise vorticity in Fig. 9a is caused by the interaction between the accelerating fluid on the periphery of the counter-rotating vortex pair (CVP), produced by the vortex generator, and the low-momentum fluid near the stable focus point A. The recirculation vortex downstream from the protrusion, shown in Fig. 9b, is caused by similar phenomena whereby the accelerating fluid particles in the flow core interact with the slow fluid in the protrusion wake and generate a transverse vorticity centering on focus point B.

The lowest values of the vorticity flux $J_z$ in Fig. 8 are obtained for the Alg–Inv–V1 (d) configuration. In fact, the distance of the tab from the HEV wall reduces the pressure gradients between the wake and the flow core, thus reducing the streamwise vorticity magnitude. From Fig. 8, it is observed that $J_z$ decreases sharply at $z/\lambda = 0.8$.
L = 0.96 for all the configurations except the Alg–Inv-V1 (d), where it seems that the counter-rotating vortex pair persists downstream from the vortex generator for a longer distance than in the other configurations. However, with its very much decreased intensity it yields a lower performance, as previously mentioned.

It is known that the increase in the vorticity flux $J_z$ enhances heat and mass transfer by increasing the convective transfers among the different flow regions [7,8]. However, the turbulent mixing is in great part caused by velocity fluctuations, and hence can be characterized by the turbulent kinetic energy $k$. The fluid mixing mechanism by velocity fluctuations, called meso-mixing, is an important process in using chemical reactions, since it determines the environment for micro-mixing, a mixing process in which reagents are mixed on scales near the Kolmogorov micro-scale [40,41].

Fig. 10 shows the axial variation of the total turbulent kinetic energy $k_{\text{total}}$ computed by the surface integral of local values of $k$ in the 7th vortex generator array:

$$k_{\text{total}} = \frac{\sum_{i=1}^{n} k dS}{\sum_{i=1}^{n} dS}$$

(16)

When the flow encounters the leading edge of the vortex generator, a shear layer is formed due to the velocity gradient between the flow core and the region below the vortex generator. It was shown [13] that this shear layer undergoes high-velocity fluctuations and thus has high turbulent kinetic energy. This phenomenon is observed in the variation of $k_{\text{total}}$ in Fig. 10 where $k_{\text{total}}$ increases from the leading edge and reaches its maximum slightly downstream from the vortex generator trailing edge. The decrease in $k_{\text{total}}$ further downstream is caused by the flow viscous effects that damp the velocity fluctuations. The difference in the $k$ levels, noted between the Alg–Inv-V1 (d) and the three other configurations, is caused by the flow acceleration below the vortex generator, thus reducing the velocity gradients between the wake and the flow core.

The meso-mixing homogeneity can be measured by $\sigma_k$, the ratio of the standard deviation of the turbulence kinetic energy over $k_{\text{total}}$:

$$\sigma_k = \frac{\sqrt{\sum_{i=1}^{n} (k - k_{\text{mean}})^2}}{k_{\text{total}}}$$

(17)

where $k_{\text{mean}}$ is the mean value of the turbulence kinetic energy on the HEV cross section.

In fact, $\sigma_k$ is a measure of the dispersion of $k$ relative to the total value $k_{\text{total}}$. Low values of $\sigma_k$ can signal good mixing homogeneity. The longitudinal variation of $\sigma_k$ is presented in Fig. 11 for the last row of vortex generators (HEV exit). It is observed that, in the region corresponding to shear-layer development near the leading edge of the vortex generator, $\sigma_k$ increases sharply since the shear layer occupies a small area in the cross section and hence the dispersion of $k$ values is greater. The dispersion $\sigma_k$ decreases rapidly downstream from the leading edge as the shear layer and thus the velocity fluctuations are spread in the flow cross section. A local

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**Fig. 9.** Temperature distribution and streamlines on (a) a flow cross section, and (b) a symmetry plane in the Alg–Inv–Rev–Prot configuration for Re = 15,000.

**Fig. 10.** Longitudinal variation of the total turbulence kinetic energy $k_{\text{total}}$ for Re = 15,000.

**Fig. 11.** Longitudinal variation of the relative dispersion $\sigma_k$ for Re = 15,000.
increase of \( \sigma_k \) is also observed near the protrusion due to large gradients in \( k \) near the protrusion boundaries, and thus increased standard deviation.

The local heat transfer can be characterized by the longitudinal variation of the local Nusselt number, presented in Fig. 12 for the last row of vortex generators, computed as:

\[
Nu_f = \frac{H_d}{\lambda} = \frac{\phi_k D}{\lambda(T_w - T_b)}
\]  

(18)

where \( H_d \) and \( \phi_k \) are respectively the local convective heat transfer coefficient and the local wall heat-flux density at the HEV cross section at axial position \( z \).

The effect of the protrusion, for the Alg–Inv–Rev–Prot (m) configuration, on the heat transfer is clearly highlighted in Fig. 12 by the local sharp increase (130% over the Alg–Inv (a) reference configuration) in the Nusselt number at the leading edge of the protrusion. Another maximum is observed at the trailing edge of the protrusion where the presence of the transverse vortex B, indicated in Fig. 5b, increases the local heat transfer by about 80% relative to the Alg–Inv (a) reference configuration, by reduction of the thermal boundary-layer thickness. The Nusselt number decreases slowly downstream from the protrusion towards the leading edge of the vortex generator. From this position on, the counter-rotating vortex pair begins to develop, as it can be seen in the vorticity flux variation in Fig. 8, and thus an increase in \( Nu_f \) is observed up to the trailing edge of the vortex generator, where the vorticity flux is maximum. From this position on the Nusselt number decreases as the vorticity flux decreases. In the configuration Alg–Inv–Rev (i), similar behavior of \( Nu_f \) is observed for the region near the vortex generator since it has the same shape as for the Alg–Inv–Rev–Prot (m) configuration.

In Fig. 12, the Alg–Inv (a) reference configuration, in the near-wake of the vortex generator, presents larger Nusselt number values than the other configurations with a slower decrease downstream from the tab. The configuration Alg–Inv–V1 (d) represents the lowest values of Nusselt number in which the variation is similar to the other geometries. However, for Alg–Inv–V1 (d) an inflection point in the variation of \( Nu_f \) is observed at the detachment zone. This inflection may be caused by flow acceleration under the tab caused by the detachment of the vortex generator.

4. Conclusions

Three-dimensional numerical simulations were performed to investigate heat transfer in twelve multifunctional heat exchangers-reactors. These configurations are based on the various high-efficiency-vortex (HEV) static mixers widely used in industry. Each configuration consists of a circular tube in which seven successive rows of trapezoidal vortex generators are inserted on the wall. The main difference among the different configurations is in the shape of the vortex generator. The study is conducted for turbulent vortic flow with Reynolds numbers ranging between 7500 and 15,000 and constant wall temperature \( T_w = 360 \text{ K} \).

First, a global analysis of the thermal performance of different configurations is conducted by using the \textit{thermal enhancement factor} to classify the different configurations at constant power consumption. It is shown that the addition of hemispherical protrusions between the vortex generator arrays greatly enhances the heat transfer with a small increase in the pressure drop. New correlations for the friction factor and Nusselt number obtained from the present results can be used to compare the present geometries to other heat exchangers in the literature. The best comparative representation is obtained by presenting the global Nusselt number versus the total power dissipation. This representation shows that the present configurations bring great improvements to the heat-transfer process with moderate pressure losses.

Numerical simulations show that a streamwise counter-rotating vortex pair is generated downstream from each vortex generator that is caused by the pressure difference between the flow core and the wake of the vortex generator. This counter-rotating vortex pair plays the role of internal agitator and thus enhances radial heat and mass transfer in the HEV cross section between the different regions of the flow.

To characterize the convective transfer, the absolute streamwise vorticity flux in the HEV cross section is calculated: it describes the intensity of the counter-rotating vortex pair. It is shown that the counter-rotating vortex pair starts to develop from the leading edge of the vortex generator and reaches maximum intensity near the trailing edge.

The total turbulence kinetic energy \( k_{total} \) begins increasing from the leading edge of the vortex generator where the shear layer is generated by the velocity gradients. The total turbulence kinetic energy \( k_{total} \) reaches its maximum near the trailing edge of the vortex generator due to the decrease in the shear layer intensity by viscous forces.

Finally, the longitudinal variation of local Nusselt number computed on different HEV cross sections shows the advantage of using protrusions between two successive vortex generator rows, since they increase the local heat transfer by increasing the temperature gradients and vorticity very close to the heated wall.

References


