ABSTRACT
Laser Doppler velocimetry is used to investigate the velocity spectra and turbulence length scales in a turbulent vortical flow. The turbulent vortical flow is ensured by vorticity generators (VGs) inserted into a straight circular pipe. Each VG generates a complex flow that is mainly the combination of a steady streamwise counter-rotating vortex pair and a periodic sequence of hairpin-like structures caused by the Kelvin-Helmholtz instability in the shear layer ejected from the VG trailing edges. These primary structures induce a secondary vorticity in the wake of the VG. The aim of the study is to analyze the velocity spectra and turbulent length scales for the different coherent structures in the flow. Thus, the Kolmogorov and Taylor microscales, the Liepmann-Taylor microscale and the viscous length scale are determined in different locations in the VG streamwise direction. The evolution of the length scales with respect to the Taylor-Reynolds number is compared with theoretical trends in a variety of flows in the open literature.

INTRODUCTION
Vortical flows can be generated either by local pressure gradients, as in turbulent wakes, or by shear instabilities, such as the Kelvin-Helmholtz instability, in free shear and turbulent jets. The large-scale coherent structures that are formed can play an important role in the mechanisms of heat, mass, and momentum transport in turbulent flows. The dynamics and production mechanisms of these coherent structures are the subject of numerous studies in the open literature [1][2][3].

The present study focuses first on the determination of turbulent length scales in a vortical flow artificially produced by a vortex generator. The results are compared to other turbulent flow configurations from the open literature and to theoretical results. In the second part, the mixing transition concept proposed by Dimotakis [4] is investigated using the present experimental results. To this end, coherent structures are generated by inserting into a straight circular pipe an array of vorticity generators (VGs) that are four diametrically opposed trapezoidal tabs inclined to the wall. Each VG generates a complex vortical flow, as shown in FIGURE 1, which dramatically enhances heat and mass transfer in the flow section [5][6][7]. A counter-rotating vortex pair (CVP) is formed in the wake of the VG because of the pressure difference between the low-momentum region under the VG and the high-momentum region above the VG in the flow core. Moreover, when the flow encounters the mixing tab, a three-dimensional shear layer forms around the tab that becomes further unstable and generates hairpin-like structures, a phenomenon caused by Kelvin-Helmholtz instability in a free shear flow. Primary hairpins can further generate secondary instabilities in the form of reverse vortices in the tab wake [6].

Instantaneous velocity measurements and spectral analysis are carried out using laser Doppler velocimetry (LDV). The statistical paths of the hairpin and reverse vortices are determined from the inflection points in the streamwise velocity profiles. The turbulence spectra and the different turbulent length scales are determined in these locations, and compared to theoretical predictions.
NOMENCLATURE

- \( C(t) \): Autocorrelation function
- \( CVP \): Counter-rotating vortex pair
- \( D \): Pipe diameter
- \( E \): Energy spectrum
- \( h \): Tab height from the wall
- \( k \): Wave number
- \( L \): Taylor macroscale
- \( R \): Pipe radius
- \( Re \): Reynolds number, \( = WD/\nu \)
- \( Re_\lambda \): Taylor-Reynolds number, \( = w\lambda/\nu \)
- \( VG \): Vortex generator
- \( w \): Axial fluctuating velocity
- \( W \): Axial mean velocity
- \( x, y, z \): Cartesian coordinate system

Greek symbols

- \( \alpha \): Inclination angle of the vortex generator
- \( \varepsilon \): Turbulent dissipation rate
- \( \eta \): Kolmogorov length scale
- \( \lambda \): Taylor microscale
- \( \lambda_v \): Inner viscous scale
- \( \zeta \): Liepmann-Taylor length scale
- \( \mu \): Micromixing length scale
- \( \nu \): Kinetic viscosity

EXPERIMENTAL SETUP

The test section consists of a straight circular pipe of 20 mm inner diameter in which one row of four diametrically opposed mixing tabs are fixed. The tabs are inclined 30° with respect to the tube wall, as shown in FIGURE 2. The test section is preceded by a preconditioner (200 cm straight Plexiglas pipe) in order to generate a fully developed turbulent flow at the test section inlet, and is followed by a postconditioner (20 cm straight Plexiglas pipe). The connections between the different elements carefully avoid any protuberance that could disturb the flow. A pulsation damper is added to the circuit to limit the pressure fluctuations produced by the pump.

FIGURE 1. THE MAIN FLOW STRUCTURES GENERATED BY A TRAPEZOIDAL VORTEX GENERATOR [7]

FIGURE 2. DIMENSIONS OF THE VORTEX GENERATOR AND THE CARTESIAN COORDINATE SYSTEM

Measurements are made using a Dantec laser Doppler velocimetry (LDV) system equipped with a 10 W argon-ion laser source and two BSA-enhanced signal-processing units (57N20 BSA and 57N35 BSA enhanced models); the measurement head is equipped with a 160 mm focal lens. The measuring volume of the LDV is positioned with a three-dimensional lightweight precision traversing system controlled via a PC.

The statistical convergence criteria are achieved on the velocity fluctuations and the mean velocity. The measurement time ranges between 60 s and 360 s, which is about 10^4 times the integral time scale, thus ensuring statistical convergence. The associated validated sampling particle number of 30,000 is needed to obtain statistical convergence. The data-acquisition rate is about 500 Hz.

LDV system calibration, including Bragg-cell oscillation and orientation sensitivity, light beam power, reference point, and optics alignment, was performed. The precision error was estimated at about 2.5%. Moreover, to ensure the reproducibility of LDV measurements, experiments were iterated four times for radial profiles at different positions for the highest Reynolds number, 15000. The relative standard deviation for the mean and RMS velocities depends on location in the measurement volume: it reaches 10-12% in the near-wall region and in the shear layer, a low-velocity, high-turbulence-intensity zone, and is about 2.5% in the flow core region. The global mean standard deviation does not exceed 6% for the mean velocity or for the turbulent fluctuations. Thus, the uncertainty can be calculated as \( \frac{2.5^2 + 6^2}{2} = 6.5\% \).

Using the sampling rate uncertainty method of Benedict and Gould [8], the confidence level is determined to be 95%; in addition, 15% of the measured data had an error less than 1% and 85% lay between 1% and 10%.

The integral time scale can be evaluated from the temporal correlation function, which requires velocity measurements with only a single probe. Measurements must be performed at
sufficiently short time intervals to detect high-frequency fluctuations. Laser Doppler velocimetry allows such fast measurements by optimizing seeding and optical adjustments. However, since LDV measurements are not performed at constant time intervals, the data were resampled according to a method suggested by Høst-Madsen and Caspersen [9]. Measurements are taken on radial profiles in the symmetry plane of one mixing tab, at 3, 6, 10, 25 and 35 mm downstream from the tab trailing edge.

STATISTICAL PATH OF COHERENT STRUCTURES

Radial profiles of the mean streamwise velocity are plotted for different axial locations in FIGURE 3 for two Reynolds numbers 7500 and 15000.

A steep gradient is observed in the mean streamwise velocity profiles at the shear zone $0.4 < y / R < 0.6$. The presence of the hairpin head in the shear layer is indicated by the upper inflection point $\left( \partial^2 W / \partial y^2 = 0 \right)$, which can be analytically computed by a polynomial fitting, following Yang et al. [10]. Thus the solid red line in FIGURE 3 coincides with the statistical path of the primary hairpin heads, showing that they move away from the wall downstream from the tab. A lower inflection point appearing below the hairpin heads denotes the presence of the reverse vortices. PIV measurements by Yang et al. [10] and DNS by Dong and Meng [6] have shown that these secondary structures have opposite vorticity and are convected downstream below the hairpins before vanishing. These inflection points are relevant in analyzing the effect of the flow structures on turbulent mixing without transient modeling.

From FIGURE 3 it can be noticed that the axial and radial development of the statistical path of the coherent structures does not depend on Reynolds number for the range studied, $7500 < Re < 15000$.

It was shown by Habchi et al. [7] that both primary hairpin heads and reverse vortices participate in the meso-mixing in the tab wake since they coincide with the spatial maxima of the turbulent kinetic energy.

ONE-DIMENSIONAL SPECTRUM

The dimensionless energy spectra $E$ for different axial locations $z/h$, obtained on the upper and lower inflection points identified in the previous section are presented in FIGURE 4, and are compared with energy spectra taken from various flow fields, such as wake behind cylinder, boundary layer, channel centerline, grid, shear layer and pipe flow. The energy spectra are normalized by $\varepsilon u_v$ where $u_v = (\varepsilon \nu)^{1/4}$ is the Kolmogorov characteristic velocity.

The Kolmogorov microscale $\eta = (\nu / \varepsilon)^{1/4}$ is here computed with $\varepsilon = 15 \nu (\partial w / \partial x)^2$, which is the turbulence energy dissipation rate obtained using Taylor’s hypothesis, as detailed by Habchi et al. [7] and Mohand Kaci et al. [13].

It can be observed from FIGURE 4 that at small scales, for $\kappa \eta > 1/8$, the spectra of all the flow fields are superimposed, as predicted by Kolmogorov’s first similarity hypothesis. In the inertial range, i.e. in the turbulent cascade, the spectra follow a $-5/3$ power-law. The energy spectra for hairpin and reverse vortices are located between the energy spectra of the wake behind a cylinder and the homogeneous shear flow. This can be explained by the fact that these two coherent structures are the combination of shear instability and wake flow behind the VG.
In the next section the determination of the different turbulent length scales is discussed.

### LENGTH SCALES ANALYSIS

The longitudinal integral length scale $L$, which represents the characteristic scale of large eddies, can be obtained from the temporal autocorrelation function on the axial fluctuating velocity $C(t)$:

$$L = \int_0^{\infty} C(t) dt$$

where $t$ is the time measured at one point.

The longitudinal Taylor microscale $\lambda$ is obtained theoretically from the second derivative of $C(t)$ at $t = 0$. After some manipulation, $\lambda$ can be expressed as:

$$\lambda = \sqrt{\frac{\sigma^2}{\langle \partial w/\partial z \rangle^2}}$$

where $\langle \partial w/\partial z \rangle = (1/|U|) \langle \partial w/\partial t \rangle$ following Taylor’s hypothesis, with $U$ the convective velocity [7].

The Taylor-Reynolds number $Re_\lambda$ is defined by:

$$Re_\lambda = \frac{w\lambda}{\nu}$$

FIGURE 5 shows the ratio $L/\eta$ as a function of the $Re_\lambda$ for the present results, compared to other recent data from the open literature collected by Tsuji [11], various experimental data and DNS for homogenous turbulence. The series are fitted by power laws recapitulated in TABLE 1.

The present results are in keeping with the theoretical 3/2 power law $L/\eta \propto Re_\lambda^{3/2}$ for homogenous isotropic turbulence, while DNS and other experiments are fitted using $Re_\lambda^{3/7}$. However, it can be observed from FIGURE 5 that the present experiments are close to grid turbulence results rigorously following the 3/2 power law. On the other hand, a bias in observed for the DNS [11] results with respect to the theoretical trends.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$L/\eta = 0.243 Re_\lambda^{3/2}$</td>
<td>$L/\eta = 0.257 Re_\lambda^{3/7}$</td>
<td>$L/\eta = 0.128 Re_\lambda^{3/7}$</td>
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FIGURE 4. ONE-DIMENSIONAL ENERGY SPECTRA FOR $Re = 15000$ AT INFLEXION POINTS (ADAPTED FROM TSUJI [11] FOR OTHER FLOW FIELDS)

FIGURE 5. LONGITUDINAL INTEGRAL SCALE NORMALIZED BY KOLMOGOROV LENGTH SCALE
This expression agrees with the theoretical trend \( \lambda / \eta \propto \text{Re}^{0.5} \) independently of the flow field conditions. In fact, the Taylor microscale represents the beginning of the viscous-dissipation range, since it is universally related to \( \text{Re}_\lambda \) whatever the flow field and the boundary conditions; this feature leads to the mixing transition concept proposed by Dimotakis [4].

**TABLE 2. POWER-LAW DEPENDENCY OF \( \lambda / \eta \) VS \( \text{Re}_\lambda \)**

<table>
<thead>
<tr>
<th>Present results</th>
<th>Experiments and DNS [11]</th>
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<tbody>
<tr>
<td>( \lambda / \eta \approx 1.968 \text{Re}_\lambda^{0.5} )</td>
<td>( \lambda / \eta \approx 1.990 \text{Re}_\lambda^{0.5\text{ext}} )</td>
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**MIXING TRANSITION**

Dimotakis [4] has demonstrated that a mixing transition occurs for Taylor-Reynolds numbers \( \text{Re}_\lambda > 100 \). On the basis of Kolmogorov’s [15] theory of locally isotropic turbulence, Dimotakis [4] studied the decoupling of the lower boundary of the inertial sub-range (or the upper limit of the viscous subrange), \( \lambda_v \), defined by [16]:

\[
\lambda_v = 50 \eta \quad (4)
\]

and the laminar length scale \( \zeta \), i.e. the Liepmann-Taylor microscale, corresponding to the upper boundary of the inertial sub-range and marking the boundary of the energy-containing spectral domain and the inertial range.

Following Dimotakis [4], the Liepmann-Taylor scale \( \zeta \) can be related to the outer scale \( \delta \) of the turbulence production domain by:

\[
\frac{\zeta}{\delta} = 5 \text{Re}_{\lambda}^{1/2} \quad (5)
\]

Dimotakis [4] claims that when the Liepmann-Taylor microscale \( \zeta \) exceeds the inner viscous scale \( \lambda_v \), a transition occurs to an enhanced mixing turbulent flow state. Fellouah and Pollard [14], studying the mixing transition downstream of a jet flow, have observed that this transition is accompanied by the disappearance of the inertial sub-range.

Moreover, the Kolmogorov scale can be written as [4][14]:

\[
\frac{\eta}{\delta} \approx \text{Re}_{\delta}^{-3/4} \quad (6)
\]

Combining Eqs. (5) and (6), one can obtain the relation between Liepmann-Taylor and Kolmogorov length scales:

\[
\frac{\zeta}{\eta} \approx 5 \text{Re}_{\delta}^{1/4} \quad (7)
\]

**FIGURE 7. LENGTH SCALES NORMALIZED BY KOLMOGOROV LENGTH SCALE**

Considering the decoupling between the Taylor microscale \( \lambda \) and micromixing scale \( \mu \) defined by Baldyga and Bourne [17] which is:

\[
\mu = 17.24 \eta \quad (7)
\]
it can be observed that for $Re_x > 75$, the micromixing scale becomes larger than Taylor microscale. It can be concluded that for $Re_x$ larger than 75, the micromixing process—namely engulfment mechanism at Kolmogorov scale—includes mixing on the scales larger than the Taylor microscale.

**CONCLUDING REMARKS**

Laser Doppler velocimetry is used to investigate the velocity spectra and turbulence length scales in a turbulent vortical flow. This vortical flow is generated by using vorticity generators (VG) inserted in a straight circular pipe. The statistical path of coherent structures is first determined from inflection points in the axial mean velocity profiles.

The energy spectra are determined for the statistical path of these coherent structures and it is found that the dimensionless spectra of the present flow lie between those in the literature for homogeneous shear flow and for the wake behind cylinder. This result is explained by the fact that the coherent structures in the present flow are generated from the combined effect of shear flow and the wake behind the VG.

Further, the variation of the Taylor macroscale with $Re_x$ shows large scatter for various flow fields and thus cannot be used to present a universal scale for the motion of the large eddies.

However, Taylor microscales obtained in the present study in hairpin and reverse vortices show good agreement with theoretical predictions and with results for other turbulent flows in the open literature, confirming that the Taylor microscale is a universal parameter that characterizes the motion of the small eddies, as stated earlier by Tsuji [11].

Finally, this study confirms that the mixing transition concept proposed by Dimotakis [4] is applicable for flows past a VG, and could further be used to design intensified reactors and mixers for various industrial processes.

**REFERENCES**


