Regimes of Passive Mode-Locking of Fiber Lasers

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Abstract: We present results of a numerical simulation and analysis of various regimes of passive mode-locking of fiber lasers including a single pulse and multipulse operation, bound states of solitons, and harmonic passive mode-locking. Our results on the multipulse regimes consist of the multihysteresis dependences of a number of pulses in the laser cavity, of pulse peak intensities and an intracavity radiation energy on a pump power. The analysis of mechanisms of an intersoliton interaction in the laser cavity has been performed. The opportunity of the coding of information with the use of bound soliton sequences has been demonstrated. Various mechanisms for control of intersoliton interactionr are proposed.

Keywords: dissipative soliton; passive mode-locking; fiber laser

1 Introduction

Lasers generating ultrashort optical pulses are widely employed in diversified areas of science, technology, and engineering [1,2]. The great diversity of applications of ultrashort pulse lasers calls for further development and perfection of this type of quantum generators. At the present time, one of main ways for creation of perfect ultrashort pulse sources is related to passive mode-locked fiber lasers [3–6]. These lasers have unique potentialities. They are reliable, compact, flexibility, low cost. The nonlinear losses based on the nonlinear polarization rotation technique are fast, practically inertia-free. For them the depth of the modulation and the saturating intensity are easily controlled through the orientation angles of intracavity phase plates. The great variety of operating regimes is an important feature of this type of lasers. They can operate either with a single pulse in the laser cavity or in a multiple pulse regime. The latter is connected with the effect of a quantization of intracavity lasing radiation into individual identical solitons [7–9]. Lasers operating in multiple pulse regimes demonstrate multistability: the number of pulses in an established operation depends on initial conditions [8–9]. Analogical regimes are realized in lasers with another mechanisms of nonlinear losses (semiconductor saturable absorber mirror, saturable absorbers based on quantum dots, carbon nanotubes, graphene and so on [10]).

The type of a soliton interaction plays a crucial role in the established multiple pulse regimes of fiber lasers. In the case of pulse attraction, bound solitons structures can be formed. Such structures were theoretically and experimentally investigated by many authors [11–16]. Possibility of a realization of strong bonds between solitons ($\sim 10\%$ of an individual soliton energy) was found in the paper [16]. As this takes place, steady-states of pair interacting solitons form a two soliton molecule with a set of energy levels corresponding to various types of bonds between pulses. With a use of this effect the high-stable noise-proof information sequences of bound solitons can be realized. In such sequences a high-density coding of the information is realized through various distributions of different energy bonds along the soliton chains.

A long-range mechanism of repulsion of ultrasort pulses results in the regime of harmonic passive mode-locking [17–21] (the regime of a multiple pulse generation in which distances between all neighboring pulses take the same value). The harmonic passive mode-locked fiber lasers are of great interest as ultrashort optical pulse sources with a high repetition rate which are employed in high-speed optical communications. This lasing regime can be also realized on basis of a sequence of bound solitons with a single type of a bond between neighboring solitons which fills completely a total laser resonator. In this case the expected rate of repetition of ultrashort pulses in the output laser radiation is of the order of inverse ultrashort pulse duration and can lay in the terahertz frequency range for sub-picosecond pulses [20,21].

A quantization of intracavity radiation into individual identical solitos is a useful phenomenon for a creation of ultrashort pulse generators with a high rate of a repetition of ultrashort pulses. The greater number of pulses in laser cavity results in the greater rate of the repetition of pulses in output radiation. However, this phenomenon is a serious obstacle for creation of generators with high energy of individual pulses. Really, in consequence of this phenomenon an increase of pumping results in an increase in number of pulses in the laser resonator, thus the energy of a individual pulse remains approximately as before. The effective control of intersoliton interactions opens new opportunities for management of generation regimes of fiber lasers. For realization of such control it is necessary to know the properties of soliton interaction at a fundamental level. In this paper we present our results on a formation of multiple pulses regimes connected with interaction of lasing solitons through a gain medium, inertia-free nonlinear losses and a nonlinear refractive index.

This paper is organized as follows. In Sec. 2 we present the results on models of passive mode-locked lasers with uniformly distributed intracavity medium. In Sec. 3 we analyze phenomena due to lumped intracavity elements inducing powerful soliton wings. Such wings result in a strong intersoliton interaction with great bound energies. Section 4 is devoted to mechanisms of management of interaction between solitons.

2 Models of Passive Mode-Locked Lasers with Uniformly Distributed Intracavity Medium

2.1 Model with Complex Cubic Nonlinearity

2.1.1 Master Equation

In the first stage of our analysis we use the complex normalized equation with a cubic nonlinearity which describes the field evolution in a unidirectional ring laser with a uniformly distributed intracavity medium [22,23]:

$$\frac{\partial E}{\partial \varsigma} = \left(D_r + iD_i\right)\frac{\partial^2 E}{\partial \tau^2} + \left[g + \left(p + iq\right)I\right]E, \tag{1}$$

where E is the electric field amplitude, τ is the time coordinate in units $\delta t = \sqrt{|\beta_2| L/2}$ (here β_2 is the second-order group-velocity dispersion for the intracavity medium and L is the cavity length), ς is the normalized propagation distance (the number of passes of the radiation through the laser cavity), D_r is the frequency dispersion of the gain and the linear losses, D_i is the frequency dispersions of the refractive index, p is the cubic nonlinearity of the losses ($p > 0$), q is the cubic nonlinearity of the refractive index, $I = |E|^2$ is the field intensity in units $(\gamma L)^{-1}$, where γ is the dimensional nonlinear refractive coefficient related to the nonlinear index coefficient. The term g is the total amplification including the linear losses σ_0:

$$g = \frac{a}{1 + b\int I d\tau} - \sigma_0, \tag{2}$$

where the integration is carried out over the whole round-trip period, a is the pumping parameter, and b is the saturation parameter. Equation (1) is the simplest equation taking into account a frequency dispersion of gain-losses and a refractive index, a nonlinearity of losses and a refractive index, and also a saturation of an amplification.

2.1.2 Results of analysis and numerical simulation

The model of passive mode-locking based on Eqs. (1) and (2) describes only two lasing regimes which are realized after a transient process: an operation with filling totally laser resonator with radiation and a regime with single ultrashort pulse in laser cavity. The amplitude of the single steady-state pulse is described by the expression

$$E_s = E_0 \frac{\exp(i\Omega\tau - i\delta k\varsigma)}{\cosh^{1+i\alpha} \beta\tau}, \tag{3}$$

where the peak amplitude of a pulse E_0, its reverse duration β, its frequency chirp α, parameters Ω and δk are determined from a system of algebraic equations. The temporal profile is $I_s = I_0 / \cosh^2 \beta\tau$. The spectral profile of this pulse is determined by the following analytical expression [24]

$$I_\nu = \frac{\pi^2 |E_0|^2}{\beta^2} \frac{\sinh \pi\alpha}{\alpha\left(\cosh \pi\alpha + \cosh \dfrac{\pi\nu}{\beta}\right)}, \tag{4}$$

where ν is a frequency detuning from the center frequency of soliton radiation. With increasing chirp α the spectral profile changes from bell-shaped to rectangle form (see Fig. 1(a)). Figure 1(b) shows the change of the frequency chirp α on the plate $\zeta = q/p$, $\theta = D_i/D_r$. Large chirp α resulting in the rectangle spectrum is realized in the case of large values of focusing nonlinearity of a refractive index $q > 0$ and of normal dispersion $D_i > 0$ (the left upper quadrant in Fig. 1(b)).

The equation (1) has also solutions with indefinitely increasing amplitude $E \to \infty$ during a transient process which are not correct because of breakdown of the condition $pI << 1$. In this case it is necessary to take into account next terms of the expansion of the nonlinear losses in a Taylor series. For the analysis of many problems it is sufficient to use the cubic-quintic nonlinear losses.

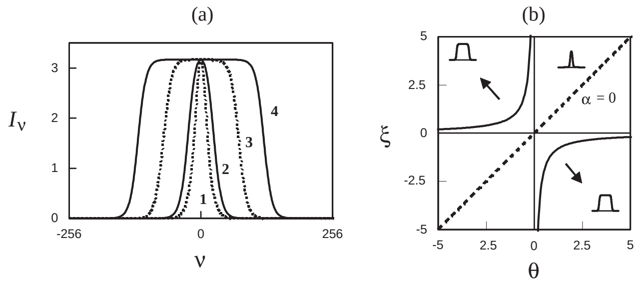

Figure 1

Spectra of ultrashort pulse described by Eq. (1). (a) Spectral profiles of established solitons with different chirps: (1) $\alpha = 0$, (2) $\alpha = 1$, (3) $\alpha = 3$, (4) $\alpha = 5$. (b) Variation of the chirp α on the plane of the nonlinear-dispersion parameters (ξ, θ). For the dashed line $\alpha = 0$, for the solid curves $\alpha = \pm\sqrt{2}$, the arrows point the directions of maximal increase in the chirp α . The spectral profiles in the figure (b) demonstrate typical spectra for the three areas separated from each other by the solid curves.

2.1.3 Some Remarks

In the case of $b = 0$, the gain is constant and the Eq. (1) is transformed into the following equation

$$\frac{\partial \Psi}{\partial t} = (d_r + id_i)\frac{\partial^2 \Psi}{\partial z^2} + \left[c + (c_1 + ic_2)|\Psi|^2\right]\Psi, \qquad (5)$$

where d_r, d_i, c, c_1, c_2 are constant parameters. This equation was obtained in the paper [26] for an analysis of hydrodynamical instabilities. This equation has also the solution in the form (3). However, there exists the principle distinction between Eq. (1) and Eq. (5). Equation (1) has the stabile solution in the form of a single stationary soliton described by Eq. (3), but in the case of Eq. (5) such stationary solution is always unstable. In the case of Eq. (1) the stabilization of the single pulse solution is realized through a saturation of the amplification g . If the parameter of the gain saturation is equal to zero $b = 0$ (see Eq. (2)) then Eq. (1) is transformed into Eq. (5). In addition, if $D_i = d_i = 0$ and $q = c_2 = 0$ then Eqs. (1) and (5) is transformed into Fisher-Kolmogorov equation [27–29].

The simplest model of a passive mode-locked laser based on Eqs. (1) and (2) adequately describes many properties of lasing ultrashort pulses. However, this model does not describe multipulse passive mode-locking. Such description becomes possible at the account of dependence of the nonlinearity p on an intensity I . This dependence appears at high values of peak intensity of pulses.

The solution with an infinite growth of the field intensity is due to imperfection of the model for nonlinear losses: what actually happens in real experimental systems is that the decrease in losses cannot be greater than the linear losses. In this regard the following form for the change in the nonlinear losses is more realistic

$$\delta\sigma = \frac{p}{1+|E|^2} - p = -\frac{p}{1+|E|^2}. \tag{6}$$

2.2 Passive Mode-Locked Laser with Saturable Absorber

2.2.1 Master Equation

Replacing in Eq. (1) the term pI by the term $pI/(1+I)$ (see Eq. (6)), we obtain the following master equation

$$\frac{\partial E}{\partial \varsigma} = (D_r + iD_i)\frac{\partial^2 E}{\partial \tau^2} + \left[g + \left(\frac{p}{1+I} + iq \right) I \right] E, \tag{7}$$

which describes a laser generation more adequately. The gain g is determined by Eq. (2). In the case of the small intensity $I << 1$ Eq. (7) is transoformed into into Eq. (1).

2.2.2 Results of Numerical Simulation

The typical transient evolution and the steady-state operation for passive mode-locking of a laser described by Eq. (7) is shown in Fig. 2. The temporal distribution of the intensity $I(\tau)$ is presented as the function of the number of passes ς of radiation through the laser cavity. The multiple pulse initial condition with various amplitudes of pulses models the variance of amplitudes of initial noise pulses. It might be well to point out that characteristics of individual pulses in the steady state are identical. This is the effect of a quantization of intracavity radiation into individual identical solitons. The detailed information on the dependence of number of pulses N in steady state on pump power a is presented in Fig. 3. As can be seen from this figure, the dependence of pulse number N on pumping a is a many-valued function; that is, the generation is multistable. The number of pulses in an established regime depends on initial conditions. The number of possible steady states increases with increasing pump power. Such dependence N on a is realized for both normal and anomalous dispersion D_i of refractive index.

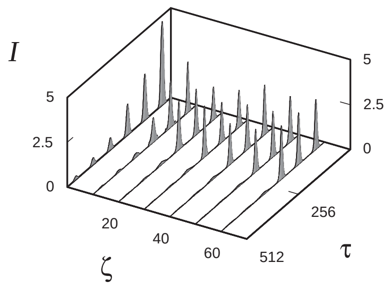

Figure 2

Transient process and steady-state multipulse operation in a passive mode-locked laser.

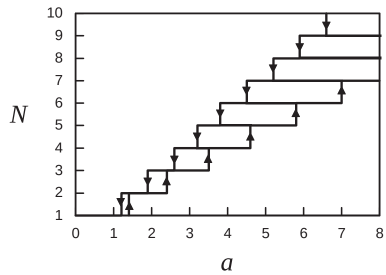

Figure 3

Multistability and the multihysteresis dependence of the number of pulses N in steady state on pump power a.

2.2.3 Discussion

The mechanism of appearance of new pulses in generation with increasing pump power and the physical nature of the quantization of radiation into individual identical solitons in passive mode-locked lasers were investigated and discussed in details in previous papers [9,25]. In the analysis of these phenomena, it is necessary to take into account the dependence of the amplification g on the total energy of the intracavity radiation (see Eq. (2)).

In the case of harmonic passive mode-locking, the multipulse generation due to the quantization of the intracavity lasing radiation into individual identical solitons is a very useful effect: the greater the number of pulses in a laser cavity, the higher the pulse repetition rate in the output radiation. In the case of creation of generators of high-energy pulses, the radiation quantization effect is a very harmful phenomenon. It prevents an increase in the energy of an individual pulse with increasing pumping. Suppression of multipulse operation for obtaining a lasing regime with a single high-energy pulse in the laser resonator provides new opportunities for increasing the pulse energy.

The authors of [6] used a model of nonlinear losses described by an additional term with a quadratic dependence on intensity. In this case, an infinite increase in intensity is suppressed due to such dependence. It was found that within the framework of this model with certain nonlinear-dispersion parameters the pulse shape became rectangular and its energy can be arbitrarily high. This phenomenon was termed a dissipative soliton resonance, and it is of interest for designing high-energy pulse lasers. Conditions for the occurrence of a single pulse and multipulse operation with increasing pump were not studied. Interaction of pulses through a common gain medium was ignored. The task about competition and coexistence of ultrashort pulses in passive mode-locked lasers under dissipative-soliton-resonance conditions with an increasing pump power is of great interest for understanding of potential of this laser regime.

2.3 Competition and Coexistence of Ultrashort Pulses in Passive Mode-Locked Lasers under Dissipative-Soliton-Resonance Conditions

2.3.1 Master Equation

For our analysis we use the following master equation describing the field evolution in a unidirectional ring laser based on the model of a distributed intracavity medium with a quadratic complex dispersion and a cubic-quintic complex nonlinearity [24,30]:

$$\frac{\partial E}{\partial \varsigma} = (D_r + iD_i)\frac{\partial^2 E}{\partial \tau^2} + \left[g + (p + iq)I - (p_2 + iq_2)I^2\right]E, \tag{8}$$

The gain is determined by Eq. (2). We analyze the regime of normal dispersion $D_i < 0$. The term pI describes nonlinear losses that decrease with increasing intensity I. The term $p_2 I^2$ is due to nonlinear losses that increase with increasing I. In the case of such nonlinear losses, the growth in the peak intensity of pulses is limited by the value $I_{\max} \sim p/p_2$. A similar limitation of the peak intensity occurs in passive mode-locked fiber lasers with nonlinear losses due to the nonlinear polarization rotation technique [9]. Equation (8) takes into account the gain saturation through the dependence of the parameter g on the energy of the intracavity radiation (see Eq. 2)). This differs Eq. (8) from the equation used in [6,28], where the parameter g is a constant.

The scalar model described by Eq. (8) gives temporal and spectral profiles of a stationary single pulse that are qualitatively similar to the corresponding results obtained using a vector model of passive mode-locking for fiber lasers [9]. At the same time, this simpler model provides a better understanding of the various features of passive mode-locking. These factors determine the use of the model (8) for the analysis of the investigated regimes of passive mode-locking of fiber lasers.

2.3.2 Results of Numerical Simulation and Discussion

Using Eqs. (8), (2), we studied the features of the passive mode-locking observed with a limitation of a decrease of nonlinear losses with increasing intensity. Numerical simulation was performed with the focusing nonlinearity q and various values of the normal dispersion D_i. Plots of the number of intracavity pulses in steady-state operation N versus pump power a are presented in Fig. 4.

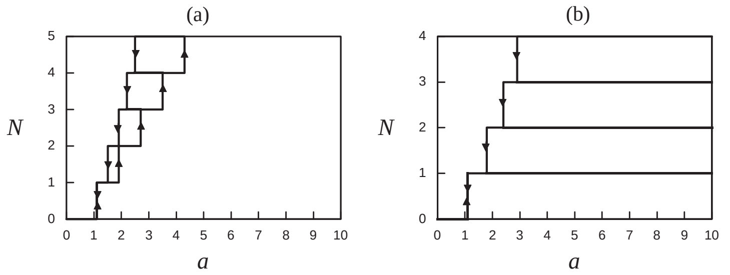

Figure 4

Number of pulses N vs. pump power a. (a) $D_i = -20$, (b) $D_i = -28$. Other parameters are $D_r = 1$, $p = 1$, $q = 18$, $\sigma_0 = 1$, $p_2 = 3$, $q_2 = 0$, and $b = 0.01$.

Increase in the number of pulses in a laser cavity with increasing pump power a is common for models of nonlinear losses taking into account that their decrease with increasing intensity I is limited (see Fig. 4(a)). However, as one can see from Fig. 4(b), in the case of a high normal dispersion D_i, this is not so. The single pulse operation regime is retained with increasing pumping. This regime is stable because the amplification g outside the volume of the pulse remains negative as the pump power a is increased.

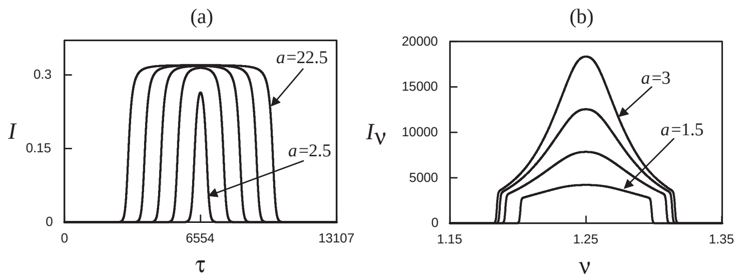

Figure 5

Dependence of the temporal (a) and spectral (b) profiles of a steady-state pulse on the pump power a. Laser parameters are the same as in Fig. 4(b).

Figure 5(a) shows that with increasing pump power a, the peak intensity of the steady-state pulse initially increases and then remains constant while its bell-shaped form is transformed into a rectangular one. Further increase in the pumping a results in a monotonic increase in the duration of the rectangular pulse. This transformation of the stationary pulse is associated with the stabilizing quadratic nonlinearity of the losses p_2 in Eq. (8). The corresponding change in the pulse spectrum is shown in Fig. 5(b). As the pump power a is increased, a bell-shaped top appears in the rectangular spectral profile (see Fig. 5(b)). Further increase in the pump power leads to the growth of the bell-shaped part of the spectrum and the rectangular spectral profile is transformed in this way to a bell-shaped one. The amplification g outside the pulse becomes negative $g < 0$, which prevents the appearance of new pulses in the laser cavity from spontaneous radiation with increasing pump power. Accordingly, with increasing pump power a, the energy of a single pulse can be arbitrarily large at a corresponding level of pumping.

In the case of dissipative soliton resonance, passive mode-locked lasers also show multistability (see Fig. 4(b)): the number of stationary pulses in steady-state operation depends on the initial conditions. Operation with initial pulses differing in duration becomes steady-state operation with identical pulses. Single-pulse operation is obtained only with a single initial pulse or with small initial pump power a. With increasing pump power, the single-pulse operation is retained.

The phenomenon of dissipative soliton resonance is due to the specific dependence of nonlinear losses on the intensity $\delta\sigma = -pI + p_2 I^2$. At low intensity, these losses decrease with increasing intensity. In contrast, at high intensity, they increase as the intensity is increased. As a result, in the case of dissipative soliton resonance, the peak intensity is stabilized at a certain level $I_{\max} \sim p/p_2$ and the pulse becomes rectangular. A similar dependence is observed in passive mode-locked fiber lasers with the nonlinear polarization rotation technique. Correspondingly, the analysis of the generation dynamics of these lasers based on the vector model also leads to rectangle pulses and the specific spectral dependence presented in Figs. 6 and 7 (see Fig. 4 in [9]). Thus, dissipative soliton resonance is a rather common phenomenon and can be observed in real lasers with passive mode-locking. The scalar model (8) adequately describes the phenomenon studied. It is significantly simpler than the vector model and provides a better understanding of the main features of the passive mode-locking process related to dissipative soliton resonance.

Smaller nonlinearities of the refractive index and larger values of the normal dispersion promote dissipative soliton resonance. This phenomenon provides new opportunities for the design of high-energy pulse lasers.

2.3.3 Some Remarks

In the case of $b = 0$ (the gain saturation is ignored), the gain is constant and the equation (8) is transformed into the Thual-Fauve equation

$$\frac{\partial \Psi}{\partial t} = (d_r + id_i)\frac{\partial^2 \Psi}{\partial z^2} + \left[c + (c_1 + ic_2)|\Psi|^2 - (c_3 + ic_4)|\Psi^4|\right]\Psi, \tag{9}$$

where d_r, d_i, c, c_1, c_2, c_3, c_4 are constant parameters. Thual and Fauve first observed stable stationary pulses in numerical experiments in the frame of this equation [30]. Here the gain saturation is ignored and stabilization of the stationary soliton is realized through the quintic nonlinearity c_3. Using Eq. (9) the authors of papers [14,15] investigated the regime of bound solitos. In the frame of this equation the effective interaction between solitons is realized only for short-range equal to several widths of an individual soliton. In the frame of Eq. (9) the phenomenon of dissipative-soliton-resonance was analyzed in the paper [6]. In this case the stationary soliton becomes be unstable. This instability is due to incorrect ignoring of the gain saturation. It is necessary to notice, that the saturation of amplification is one of primary factors which determines work of the generator.

3 Models of Passive Mode-Locked Lasers with Lumped Intracavity Elements

3.1 Passive Mode-Locked Fiber Laser with Nonlinear Polarization Rotation Technique

The type of an interaction between solitons plays a crucial role in the steady-state multiple pulse operation of passive mode-locked laser. Models with uniformly distributed intracavity nonlinear-dispersion medium demonstrates only short-range interaction between solitons which is equal to several soliton durations. Models with lumped intracavity elements shows a long-range interaction. The long-range interaction is realized by the following way.The soliton circulating in the laser cavity periodically experiences perturbations caused by lumped nonlinear losses and various intracavity components. After each perturbation, the soliton emits a dispersive wave. Constructive interference between these waves forms powerful spectral sidebands and powerful extended soliton wings. These wings result in long-range interaction and provide the formation of bound steady states of interacting solitons with a large binding energy. In fiber lasers with the nonlinear polarization rotation technique, the nonlinear losses are essentially lumped.

3.1.1 Master Equation

The laser resonator contains a polarization control system including the following sequentially arranged components: a half-wave phase plate with orientation angle α_3 with respect to the x axis, a quarter-wave plate (orientation angle α_2), a polarizing isolator (the passing axis is parallel to the x axis), and a second quarter-wave plate (orientation angle α_1) [9]. The polarization control system produces nonlinear losses that form ultrashort pulses in the laser resonator.

For our analysis of a fiber laser with the nonlinear polarization rotation technique, we use the following equations

$$\frac{\partial E}{\partial \varsigma} = (D_r + iD_i)\frac{\partial^2 E}{\partial \tau^2} + \left[G + iq|E|^2\right]E, \quad (10)$$

$$E_{n+1}(\tau) = -\eta\left[\cos(pI_n + \alpha_0)\cos(\alpha_1 - \alpha_3) + i\sin(pI_n + \alpha_0)\sin(\alpha_1 + \alpha_3)\right]E_n(\tau), \quad (11)$$

Equation (10) describes the evolution of the field in the fiber. Equation (11) connects the amplitudes of the electric field before and after the *n*-th pass of radiation though the polarization control system.

3.1.2 Multipulse Operation and Hysteresis Phenomena

The investigated passive mode-locked laser demonstrates the transient process and multihysteresis phenomena which are analogical to the presented in Figs. 2, 3. The corresponding changes in the intracavity radiation energy $J = \int I(\tau)d\tau$ and in the peak intensity of intracavity identical pulses I_0 are shown in Fig. 6.

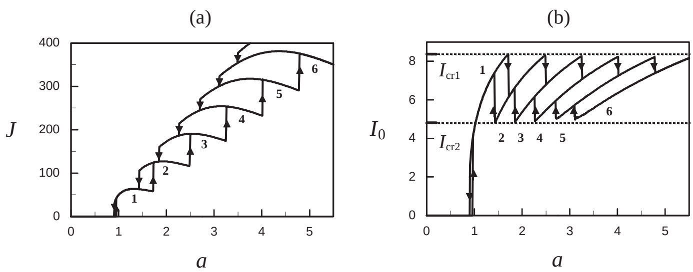

Figure 6
Multihysteresis dependences of the intracavity radiation energy J (a) and the peak intensity I_0 (b) on pumping a .

3.1.3 Bound States and Information Sequences

In this Section for our numerical simulation we have used typical parameters of Er-doped fiber laser with anomalous net dispersion of group velocity. The solitons have powerful wings which result in a strong interaction between solitons. The pair of such solitons is united in the stability formation with a large binding energy – highly-stable “two soliton molecule”. The radiation energy of such molecule is less than the energy of two solitons placed from each other on a long distance. The binding energy for two solitons in this molecule takes the discrete set of values shown on Fig. 7. Large binding energies for the low energy steady-states are due to powerful wings of solitons.

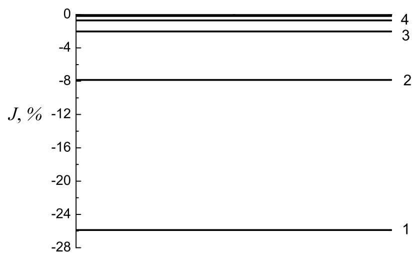

Figure 7

Binding energy of two solitons in steady-states J expressed in relative units (the binding energy divided by the energy of a single soliton).

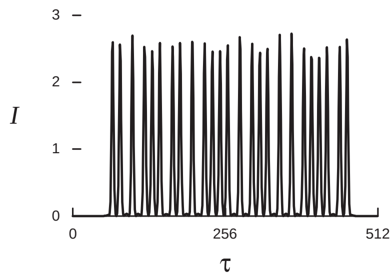

Figure 8

Stable molecule chain of bound solitons with the ground and first excited types of bonds in which the number 10122013 is coded in binary system 100110100111001100011101.

For the all odd levels, the field functions are antisymmetric $E_k(\tau) = -E_k(-\tau)$ if the origin of the coordinate $\tau = 0$ corresponds to the point equally spaced from the peaks of the solitons. In this case, the peaks amplitudes of two solitons are in opposite phase ($\delta\varphi \approx \pi$). For all even steady-states the field functions are symmetric $E_k(\tau) = E_k(-\tau)$ and the peak amplitudes of two solitons have the same phase ($\delta\varphi \approx 0$).

One of the usual way of coding the information for its transfer through optical communication fiber lines consists in the following. In equidistant initial sequences of pulses, some pulses are removed. It arises two positions (a pulse is present and a pulse is absent) which are required for the coding of the information in binary system (zero and unit). Displacement and merge of pulses in such information ultrashort pulse sequences, that is due to various types of technical perturbations including noise radiation, results in loss of the information. There are various ways of increase of a tolerance to these perturbations. Among them there is an increase in distance between the neighboring pulses in initial pulse sequence. However, this way results in the decrease in the speed of a transfer of information. In this Section we consider the nonlinear regime of propagation of pulse information sequences. The interaction of neighboring pulses results in the stabilization of this sequence. Because various types of bonds between neighboring pulses can be realized, accordingly, the coding of the information in such sequences can be realized through various distributions of types of bonds between neighboring pulses along a soliton train. Thanks to powerful wings, the binding energy for such solitons appears high, that provides the high degree of tolerance against various perturbations in the case of such sequences. Dense packing of pulses in such sequence provides high speed of transfer information. Due to large binding energies, such multisoliton molecules are highly-stable and noise-proof. Placing several initial pulses on certain distances from each other, after transient process we have obtained stationary "molecular chains" with any desirable distribution of types of bonds between neighboring solitons along a pulse train. Such sequence is realized more simply with a use of the ground and first excited types of intersoliton bonds for which the binding energies are especially great.

Figure 8 shows such information soliton sequence in which the number 10122013 is coded in binary system (10.12.2013 is the data of 3rd International Conference on Optics Photonics and their Applications – ICOPA'2013. Here the ground type of a bond (smaller distance between pulses) corresponds to unit and the first excited type of a bond (the greater distance between pulses) corresponds to zero. In binary system this sequence corresponds to the number 1001101001110011100011101, that in decimal system is the number 10122013. Really, $1\cdot 2^{24}+1\cdot 2^{23}+0\cdot 2^{22}+\cdots+1\cdot 2^{1}+1\cdot 2^{0}=10122013$.

Such soliton trains are highly stable formations. The high stability is primarily due to large binding energies. Furthermore, there exists a second reason of the high stability. It consists in the following. The perturbation energy which was initially localized in the vicinity of some pair of bound solitons is quickly collectivized among all solitons of the train. In the numerical simulation we have used the random radiation noise to prove this stability. This noise induces up to 10\% fluctuations of peak intensities of solitons but does not change the structure of soliton sequences.

3.2 Passive Mode-Locked Fiber Laser with Lumped Saturable Absorber

3.2.1 Properties of Lasing Regimes

Lumped saturable absorber can be based on various materials: carbon nanotubes, graphene, saturable absorbers based on quantum dots and so on [10]. In this case the equation Eq. (11) is replaced by the following one

$$E_{n+1}(\tau)=E_n(\tau)\exp\left(-\frac{s_{nl}}{1+pI_n(\tau)}\right). \qquad (12)$$

This equation describes the change in the field under its pass through the lumped saturating absorber, where s_{nl} is the losses for a weak field, here p is the parameter of a saturation. We have studied the formation of bound states of interacting solitons and obtained analogical results as for the case of nonlinear losses due to the nonlinear polarization rotation technique.

In this case the powerful soliton wings are also realized. These powerful wings result in large bound energies of interacting pulses. For both cases of a realization of nonlinear losses we have used sufficiently close nonlinear-dispersion parameters of the investigated laser systems.

3.2.2 Mechanism of Formation of Powerful Long-Range Soliton Wings

Large bounding energies of interacting solitons are due to their powerful wings. In this section we analyze reasons resulting in such wings. Figure 12(b) demonstrates the additional structure on the bell-shaped spectral profile of a single soliton which has the spectrum sideband form. Sideband generation in soliton spectrum is a well-known phenomenon. The sidebands result from an interference between the soliton and dispersive waves. Such dispersive waves are emitted by a soliton when it circulates in a laser resonator and periodically experiences perturbations caused by the lumped intracavity components. The interference of such wave during several circulations forms the powerful long-range wings of solitons. This mechanism does not work in the case of a continuously distributed intracavity nonlinear-dispersion medium. In this section we check the hypothesis about a formation of powerful soliton wings at the expense of dispersive waves.

We study passive mode-locked laser with the combination of the uniformly distributed saturable absorber and the lumped saturable absorber. To follow the change of properties of soliton wings due to dispersive waves and correspondingly of properties of steady-states we reduce the magnitude of the lumped saturable absorber s_{nl} up to zero. Simultaneously we increase the value of the distributed nonlinear losses σ_{nl} thus that the total losses for a weak signal for one pass of a field through the resonator remain constant. If our hypothesis is true, thus dispersive waves should weaken up to zero and the amplitude of soliton wings should decrease that will result in the change of properties of bound steady-states.

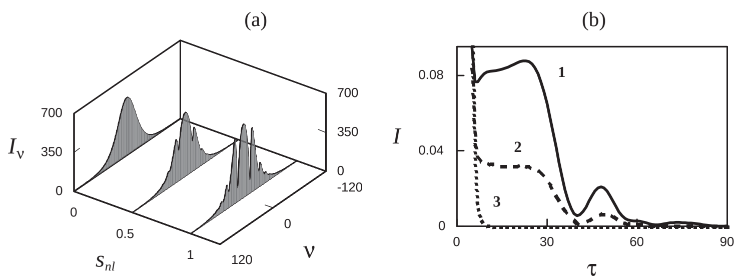

Figure 9

(a) Change in spectrum of single ultrashort pulse and (b) in its wing intensity with the change in a value of lumped part of saturable absorber s_{nl}: (1) $s_{nl} = 1$, (2) $s_{nl} = 0.75$, (3) $s_{nl} = 0$. The total value of nonlinear losses including lumped and distributed parts remains the same.

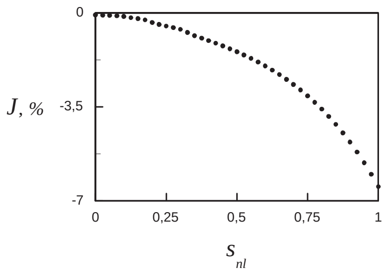

Figure 10

Bound energy for the first excited steady-state of two soliton molecule with changing value of a lumped part of a saturable absorber s_{nl}.

Figure 9(a) shows the spectral change in a single soliton with increasing lumped part of nonlinear losses s_{nl}. One can see the decrease and disappearance of sidebands in the soliton spectrum. Figure 9(b) shows the decrease of the soliton wing with decreasing lumped part of nonlinear losses. Figure 10 shows the decrease of a bounding energy for the first excited steady-state of a pair of bounding solitons with decreasing values s_{nl}. These results demonstrates the role of dispersive waves in a formation of powerful soliton wings which determine properties of bound steady-state of interacting solitons and their long-range interaction.

4 Management of Interaction between Solitons

4.1 Spectral-Selective Control of Soliton Interaction

In the paper [29] we have proposed a way to control the interaction of dissipative solitons in fiber lasers. It is based on an additional narrow spectral selection of intracavity radiation. Such selection allows us to realize the long distance wings of intracavity dissipative solitons with control of both their phases and their frequencies. As an important result, the type of interaction (attraction or repulsion) can be managed. The interaction type depends on the detuning of the central frequency of the selector from the centre of the spectral gain band. We presented the qualitative picture of peculiarities of an interaction of pulses in the investigated laser system. Among these peculiarities, there arises the relation between the spectral bandwidth of the selector transmission and the distance of interaction between pulses, the dependence of the soliton velocity with respect to the frequency detuning of the selector transmission from the centre of a gain band, and so on. The results obtained are of great interest to control the operating regime of fibre lasers through the control of the interaction of intracavity solitons, among which is the harmonic passive mode-locking. They can also be of importance to control the interaction of ultrashort pulses in fiber communications lines.

4.2 Control of Soliton Interaction by Continuous External Optical Injection

In the paper [30] we have shown by numerical simulation that the nonlinear interaction between a laser soliton and an injected monochromatic continuous wave results in their phase locking. As a consequence, the velocity of the soliton begins to depend on the amplitude and frequency of the injected radiation. It has been found that if the frequency of the external signal coincides with the frequency of the dispersive waves emitted by solitons in a laser cavity with lumped intracavity elements (see Fig. 11(a)), a mechanism for controlling long-range soliton interaction occurs. This mechanism is related to the interference between the injected wave and the dispersive waves involved in the strong long-range interaction between solitons. We have demonstrated the mechanism of soliton-soliton repulsion and, as its consequence, the occurrence of harmonic passive mode-locking (see Fig. 11(b)).

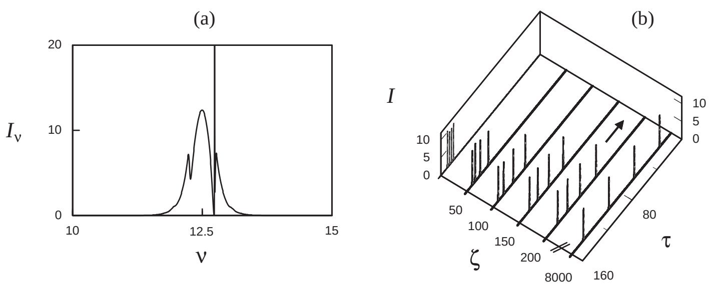

Figure 11

(a) Spectrum of an individual soliton and an injected monochromatic wave falling in a right spectral Kelly sideband. (b) Transient process and established harmonic passive mode-locking.

Conclusions

On basis of numerical simulation we have studied the basic features in a realization of single pulse and multiple pulse operation of passive mode-locked fiber lasers. It is found that the multihysteresis dependence of a number of pulses on pump results in an analogical multihysteresis dependence for the intracavity radiation energy and for the peak intensity of identical solitons. Bound steady-states of a two soliton molecule are determined. We have demonstrated the possibility to form information soliton sequences with any desirable distribution of the types of bonds between neighboring pulses along soliton trains. Thanks to large values of binding energies, such sequences have a high level of stability against perturbations. It is found that dispersive waves emitted by solitons because of lumped nonlinear losses form powerful soliton wings resulting in great bounding energy of interacting solitons in steady-states. Compitition of ultrashort pulses under dissipative-soliton-resonance conditions has been investigated. Various mechanisms for control of intersoliton interactionr are proposed.

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